Results of statistical data analysis of random road profiles

J. vom Scheidt H.-J. Starkloff R.Wunderlich

Abstract

The investigation of road vehicle dynamics has to take into account the influence of road irregularities. To do this various road profiles have been surveyed. Samples of road profile heights can be used as input of vehicle dynamics simulation or to fit appropriate random road models to the data. For supporting this modeling part the paper presents some statistical characteristics of a number of road profile samples.

1 Introduction

The paper reports the results of a statistical data analysis of some samples of measured road profile heights. First, there are six so-called IR6 samples, which have been used for the investigation of vehicle dynamics by a research group of the department of road vehicles at TU Budapest. Second, two samples used by a research group of the department of mathematics of the former TH Zwickau, are analysed. One of these two samples has been used to fit a model to the data. Monte-Carlo simulations based on this model are the source of a third group of samples.

The results presented in the subsequent sections have been obtained with the help of the statistical data analysis software ISP.

2 The samples

Table 1 lists some sample parameters as speed (of the measuring vehicle), sampling frequency and distances, sample size, profile length, and the names and a short description of the profiles.

The IR6 profiles are named by their roughness (normal, low, high, see Figure 1). It can bee seen that the six IR6 samples form three pairs - (road40[1], road80), (road65l, road95l) and

[1]The original sample contains 17.409 values. Here, only $2^{14} = 16.384$ values are used. This ensures the same sample size for the fast Fourier transformation (where zeros are added at the ends of the samples until the sample size is a power of 2) for all IR6 profiles.

name, description	speed [km/h]	sampling frequency [Hz]	sampling distance [cm]	sample size	profile length [m]
IR6-N - normal roughness					
road40	40	204.8	5.4	16.384[1]	944
road80	80	204.8	10.9	11.264	1.222
IR6-L - low roughness					
road65l	65	204.8	8.8	13.312	1.174
road95l	95	204.8	12.9	9.216	1.187
IR6-H - high roughness					
road65h	65	204.8	8.8	13.312	1.174
road95h	95	204.8	12.9	9.216	1.187
Zwickau-G - good road	80	200	11.1	4.900	544
Zwickau-B - bad road	80	200	11.1	4.900	544
Monte-Carlo M1	80	190	11.7	4.900	573
M2	80	190	11.7	16.384	1.916
M3	80	190	11.7	65.536	7.665

Table 1: Parameters of the analysed samples

(road65h, road95h) - which contain the profile heights of three different roads surveyed at two different speeds of the measuring vehicle(40/80 and 65/95 km/h). The further data analysis is based on the "low speed" samples (road40, road65l, road65h) since they contain more sample points and the sampling distances are smaller.

The terms "good" and "bad" road of the Zwickau profiles are originated historically, they cannot be motivated by the results of this data analysis.

The Monte-Carlo samples are based on a model of (auto-)correlation function (see section 4)

$$R(\tau) = \sigma^2 e^{-\gamma|\tau|},$$

where $\sigma^2 = 0.0019[m^2]$ and $\gamma = 0.054[m^{-1}]$ are chosen. These parameters result from a model fit to the Zwickau good road sample. The three Monte-Carlo samples form a sequence of (sub)samples of increasing size, i.e. M1 is a subsample of M2 and M3 contains M2 as a subsample. Thereby the M1 sample has the same sample size as the two Zwickau samples, the size of the M2 sample is similar to the IR6 profiles while the M3 profile is considerably longer then the other profiles.

3 Profile plots

A first view of the data is given by the Figures 1 to 6, where the profile heights are plotted. To compare several plots in one diagramm the profiles are shifted vertically by 0.5 m, 1 m and 1.5 m for the lower, middle and upper profile, respectively.

From Figure 1 which contains the plot of the first 1000 m of the IR6 profiles the different roughness characteristics of the three profiles can be seen. Further it can be observed that the profiles are dominated by a long wave oscillation with a wave length of 60...80 m, which contains most of the sample variance (see also the discussion of the estimated spectral density functions for these profiles in Section 4). Profile IR6-N shows an extreme minimum at 750 m.

For the Zwickau profiles presented in Figure 2 it can be observed that their length is 500 m only and that their variance is smaller. Dominating long wave oscillations are not observed.

The plots of the first 200 m (see Figure 3,4) and of the first 50 m (see Figure 5,6) show some more details of the profiles. The IR6-N profile shows some sharp downward peaks. If these peaks are no measurement errors they might be caused by potholes or grooves in the road. The 50 m-zoom further indicates, that fluctuations of the IR6 profiles start after passing a certain starting distance of about 10 m. The same effect can be observed for the last meters of the profiles (not presented). So, the first and the last values of these samples should be removed.

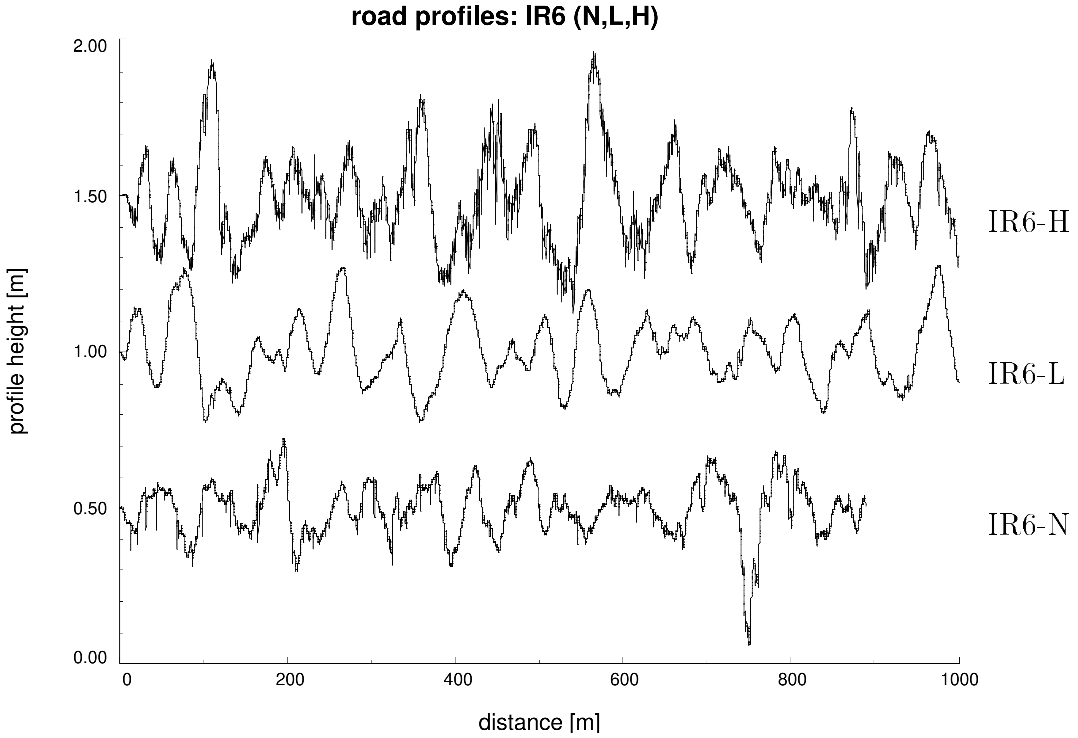

Figure 1: IR6 profiles

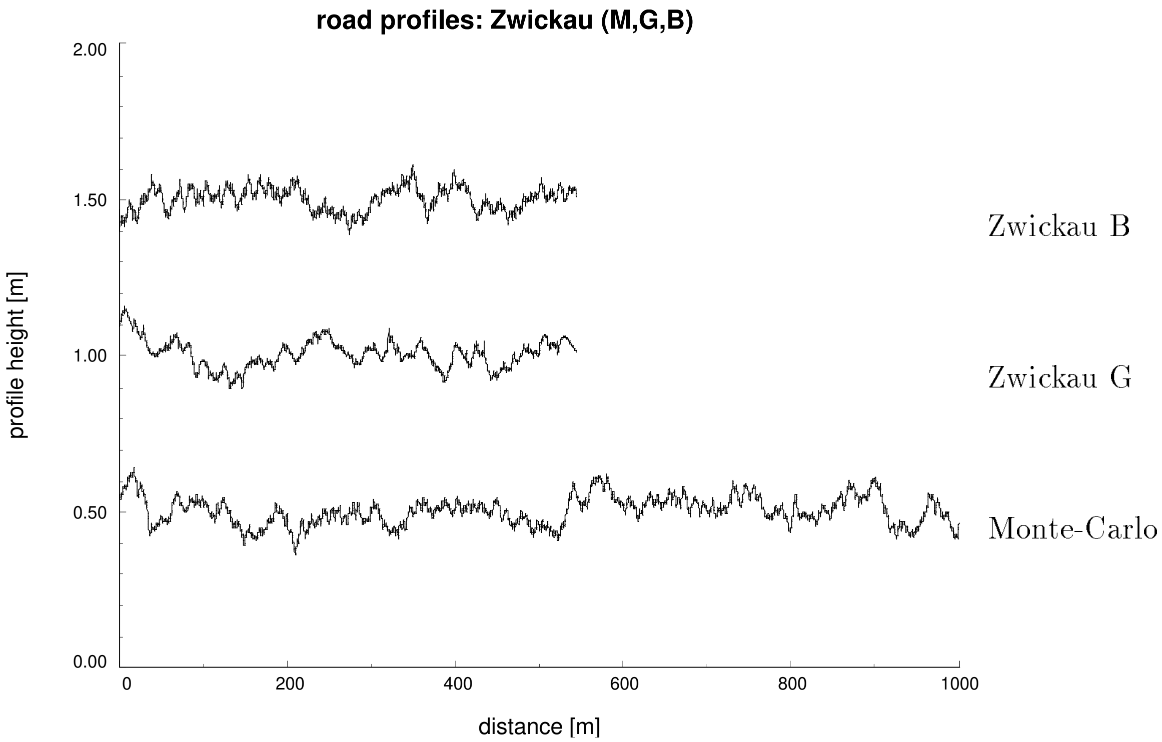

Figure 2: Zwickau and Monte-Carlo profiles

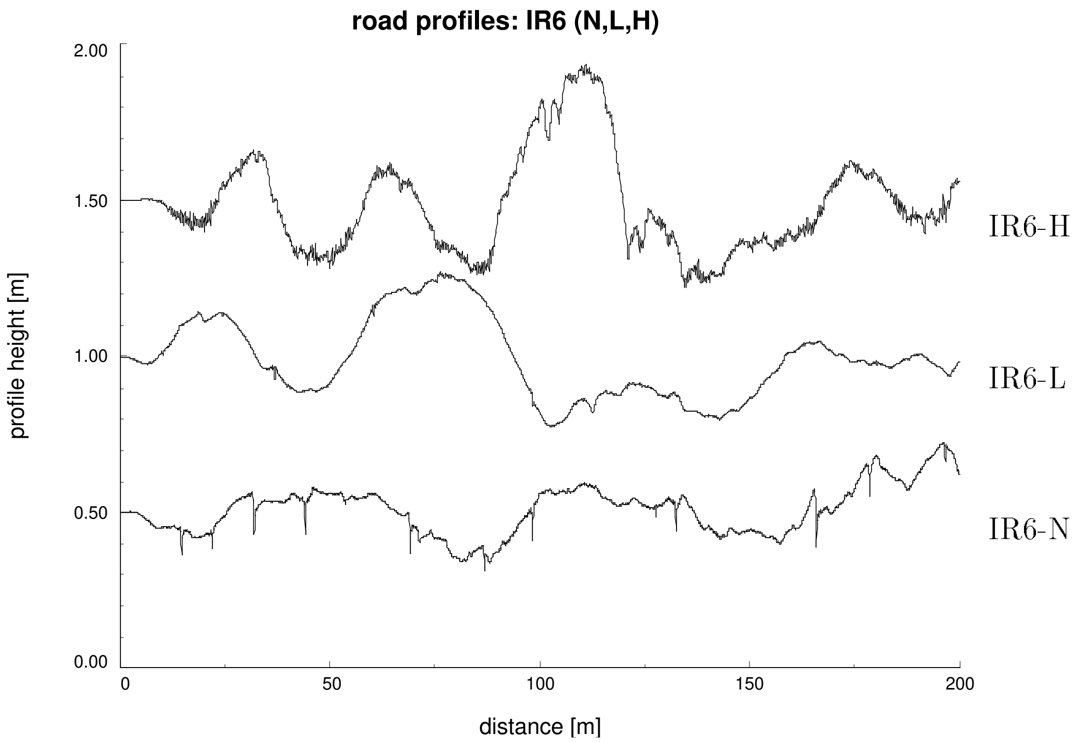

Figure 3: IR6 profiles

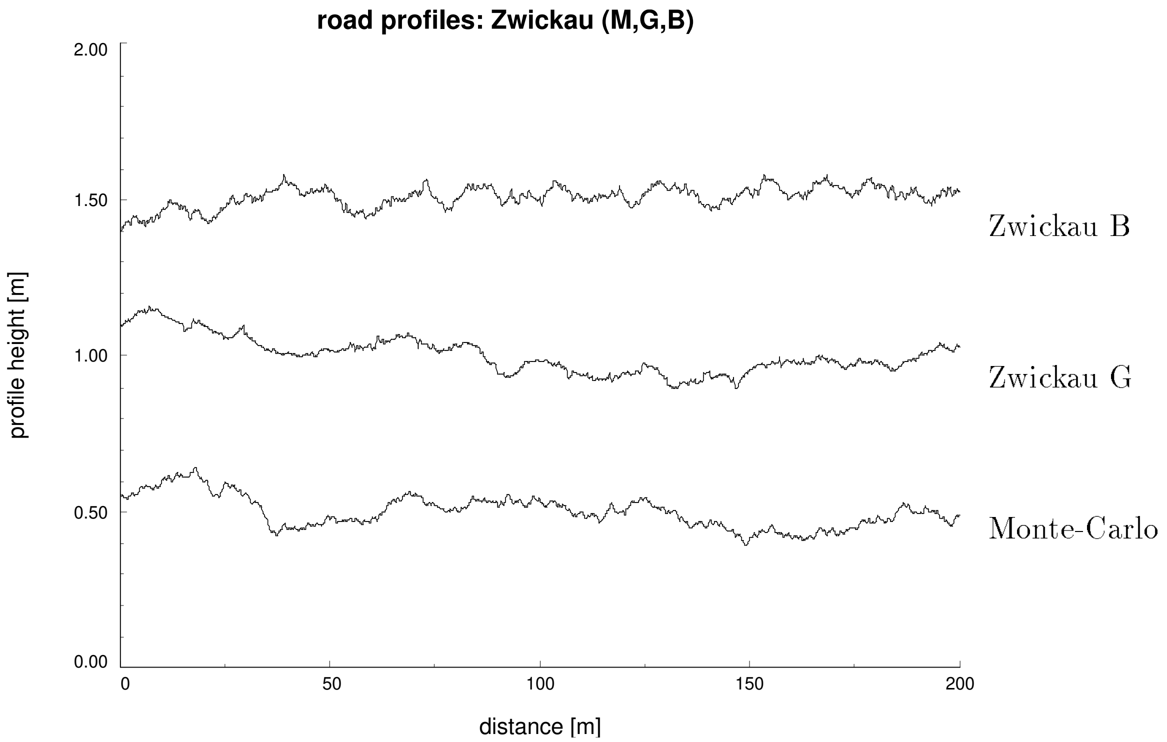

Figure 4: Zwickau and Monte-Carlo profiles

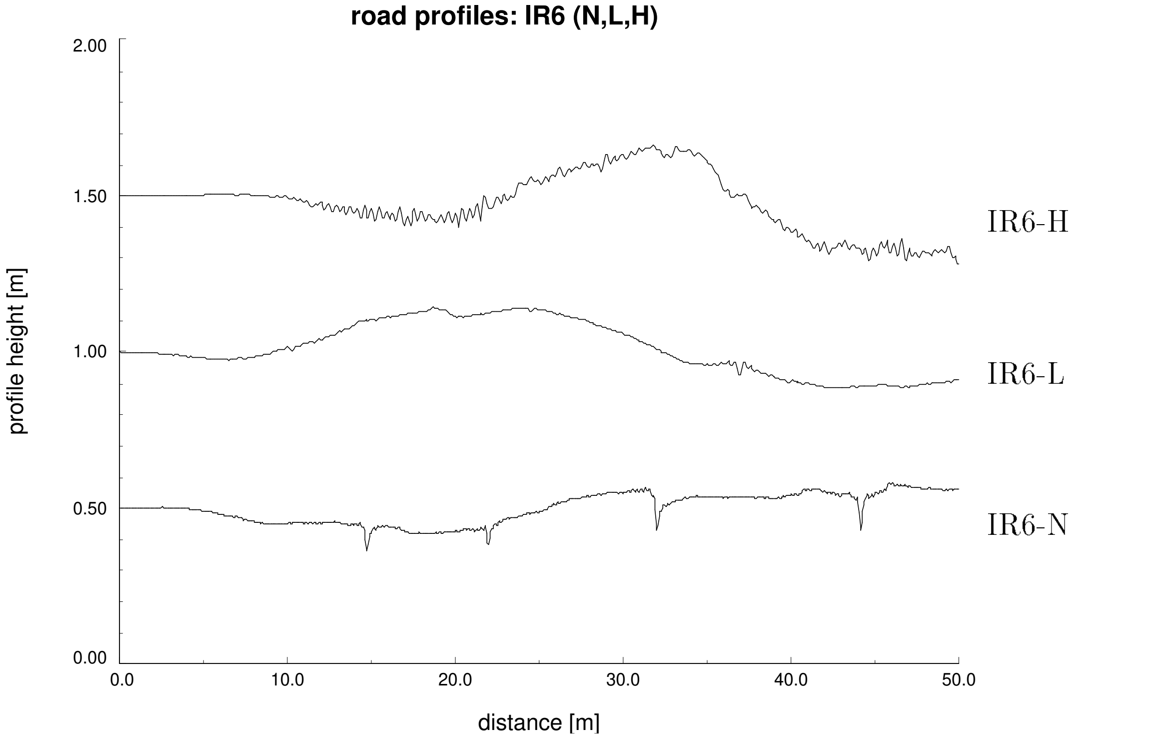

Figure 5: IR6 profiles

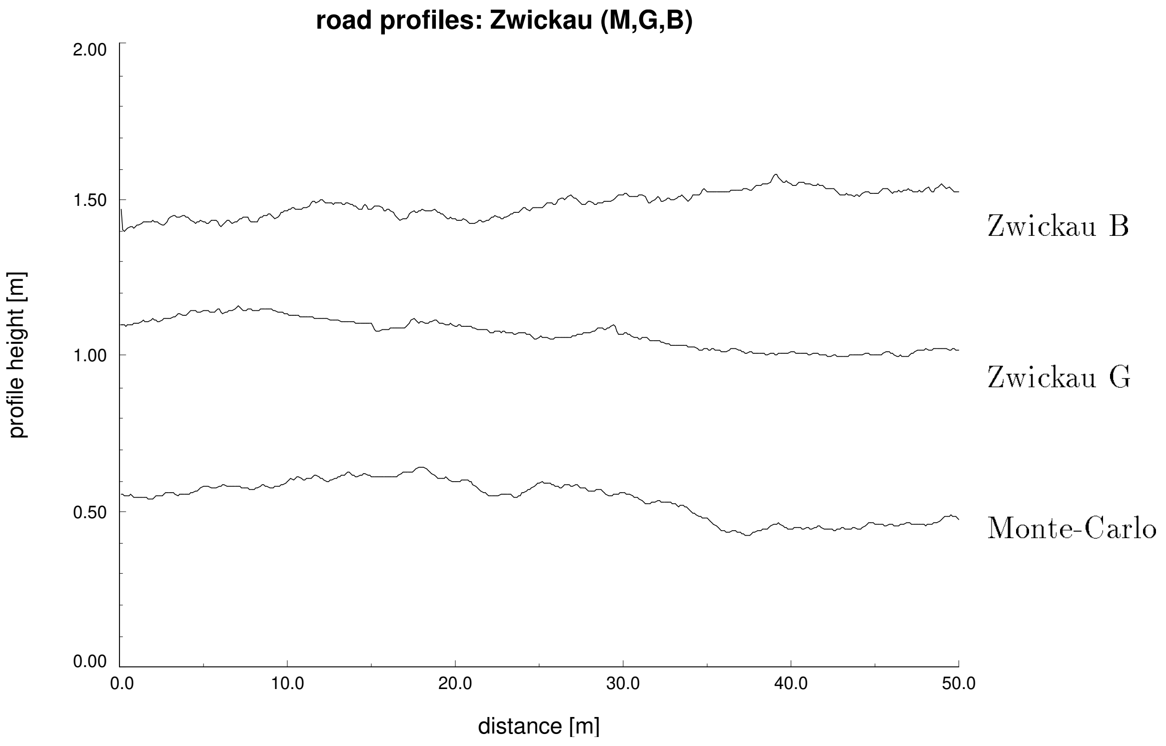

Figure 6: Zwickau and Monte-Carlo profiles

It has been mentioned that there exist two samples for each one of the IR6 profiles - a low and a high speed version. Figure 7 compares the two samples for the case of IR6-N profile - corresponding to speeds of 40 and 80 km/h - and shows a highly zoomed plot of the two profiles to visualize the differences. Non-zoomed plots show similar patterns for both profiles, only. In Figure 7 short wave oscillations can be observed which might be a result of measurement errors and not of road profile fluctuations, since their wavelength and amplitudes depend on the speed of the measuring vehicle. These observations indicate the need of sample smoothing.

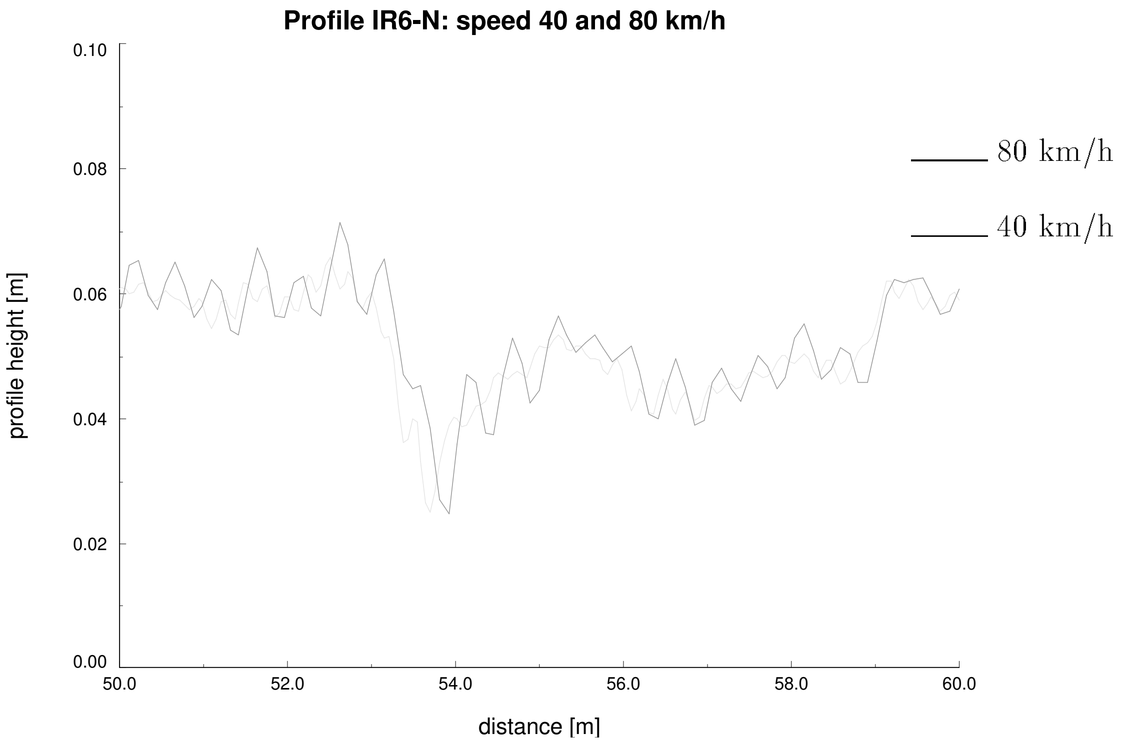

Figure 7: IR6-N profile - speed 40 km/h and 80 km/h

		IR6			Zwickau				
		N	L	H	G	B	M1	M2	M3
standard deviation	[cm]	8.6	10.6	13.5	4.5	3.8	4.1	4.3	4.3
mean	[cm]	-0.4	-0.0	-0.3	0.6	0.3	-0.6	0.0	0.3
minimum	[cm]	-44	-22	-38	-11	-11	-12	-14	-14
maximum	[cm]	22	28	46	16	11	9	14	14

Table 2: Basic statistical characteristics

Table 2 shows some basic statistical characteristics as sample mean, standard deviation, minimum and maximum calculated from the several samples. These quantities confirm the effects observed in the profile plots. Furthermore, they show, that

- sample means are approximately zero,
- the standard deviations of the IR6-L profile is larger than the standard deviation of the IR6-N profile, while the IR6-N profile shows more roughness,
- the minimum value of IR6-N profile (-44 cm) indicates the "outlier" at 750 m.

The histograms in Figure 8 and 9 visualize one-dimensional distributions of the profile height of the IR6-N and the Zwickau-G profile.

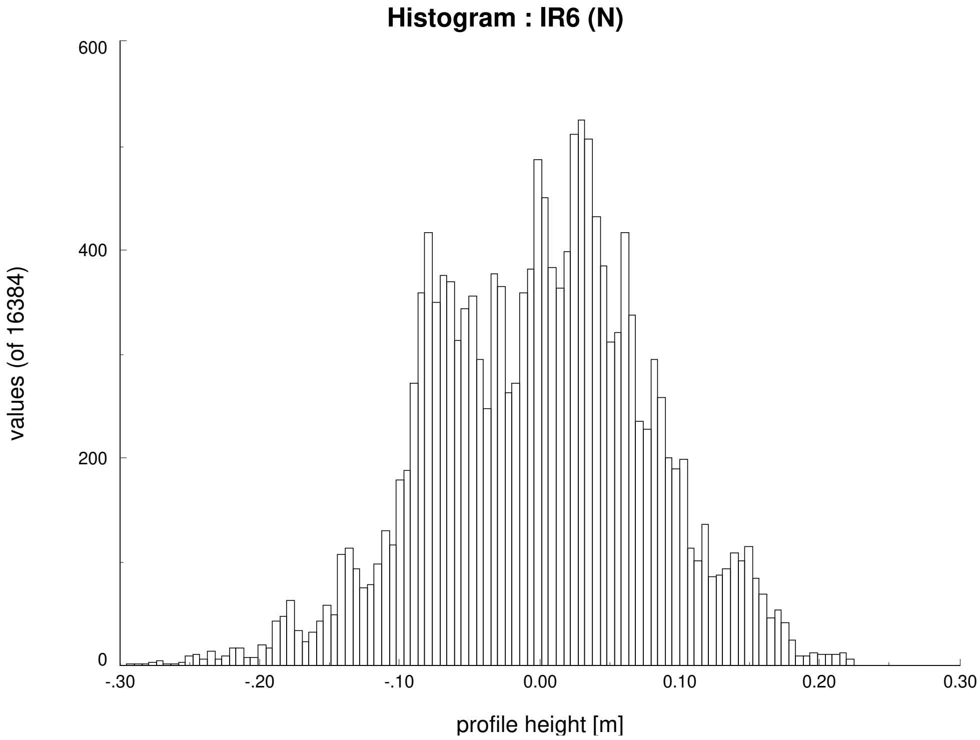

Figure 8: IR6-N profile - histogram

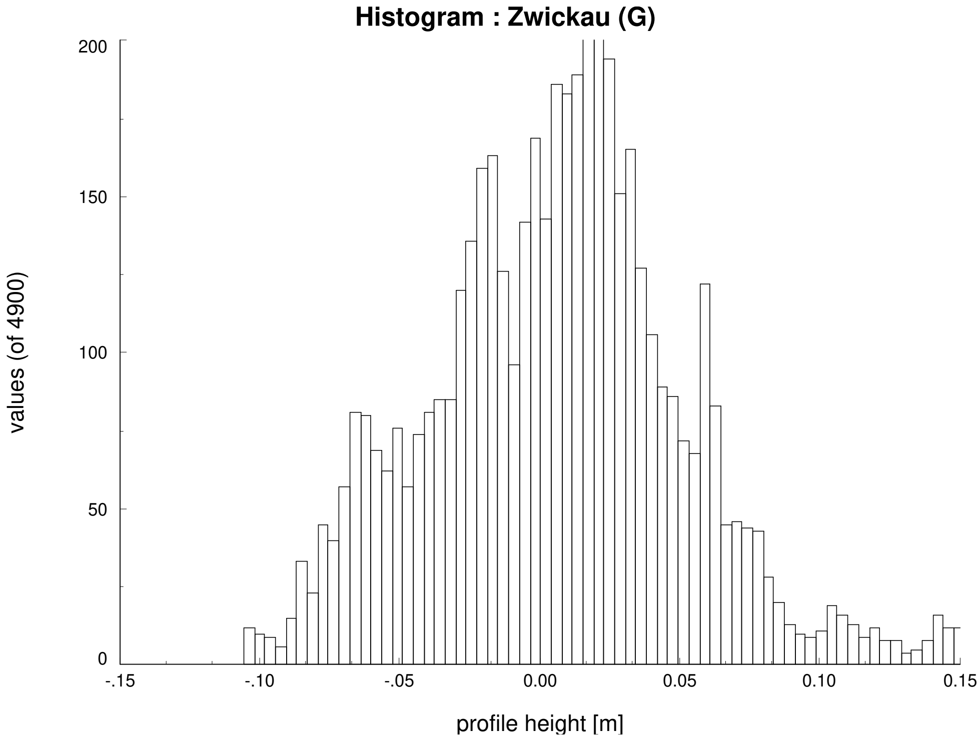

Figure 9: Zwickau-G profile - histogram

4 Correlation and spectral density function

Now we consider estimates of (auto-)correlation and spectral density functions which are widely used statistical characteristics of wide-sense stationary random processes.

Let $g(s,\omega)$, $s \in (0,\infty)$, be such a wide-sense stationary random process (here, $g(s,\omega)$ models the profile height), then it holds

$$\begin{aligned}
\mathbf{E}\{g(s)\} &= m = \text{ const} && \text{(mean)}\\
\mathbf{E}\left\{(g(s)-m)^2\right\} &= \sigma^2 = \text{ const} && \text{(variance)}\\
R(s,s+\tau) &= \mathbf{E}\{(g(s)-m)(g(s+\tau)-m)\} && \text{(correlation function)}\\
&= R(\tau) && \text{(depends only on shift } \tau)
\end{aligned}$$

where $\mathbf{E}\{.\}$ denotes expectation.

The correlation function can be used as a measure of dependence of the values of the function g at instants s and $s+\tau$.

The spectral representation of the correlation function

$$R(\tau) = \int\limits_{-\infty}^{\infty} e^{i\alpha\tau} S(\alpha)\,d\alpha = 2\int\limits_{0}^{\infty} \cos\alpha\tau\, S(\alpha)\,d\alpha$$

contains the spectral density function $S(\alpha)$, which provides information in the frequency domain. The contributions of the frequencies in an interval $(\alpha, \alpha+d\alpha)$ to the total variance σ^2 can be measured by $S(\alpha)d\alpha$. Especially it holds

$$\sigma^2 = R(0) = \int\limits_{-\infty}^{\infty} S(\alpha)\,d\alpha.$$

Table 3 presents some correlation and spectral density functions, which are used in literature (see e.g. [2]) to describe random road profiles.

	$R(\tau)$	$S(\alpha)$	$S(\alpha)$ for $\alpha \to \infty$
1	$\sigma^2 e^{-\gamma\|\tau\|}$	$\frac{\gamma\sigma^2}{\pi}\frac{1}{\gamma^2+\alpha^2}$	$O(\alpha^{-2})$
2	$\sigma^2 e^{-\gamma\|\tau\|}\cos\beta\tau$	$\frac{\gamma\sigma^2}{\pi}\frac{\alpha^2+\gamma^2+\beta^2}{(\alpha^2-\gamma^2-\beta^2)^2+4\gamma^2\alpha^2}$	$O(\alpha^{-2})$
3	$\sigma^2 e^{-\gamma\|\tau\|}(\cos\beta\tau - \frac{\gamma}{\beta}\sin\beta\|\tau\|)$	$\frac{2\gamma\sigma^2}{\pi}\frac{\alpha^2}{(\alpha^2-\gamma^2-\beta^2)^2+4\gamma^2\alpha^2}$	$O(\alpha^{-2})$

Table 3: Correlation and spectral density functions of random road profiles

Figure 10 shows the estimated correlation function of the IR6-N profile and Figure 11 compares this estimate with the estimates of the other IR6 profiles and the Zwickau profiles. The graphs of these functions show different ordinates at the origin ($\tau = 0$), which indicates different variances. Further a fast decrease (first zero of IR6 profiles at $\approx 15 \ldots 20$ m and of the Zwickau profiles at ≈ 50 m) and an oscillating behaviour of the graphs can be observed.

Comparison of these empirical results with the models of correlation functions in Table 3 leads to the proposal of model 2 or 3 which contain oscillating terms ($\cos \beta\tau$ or $\cos \beta\tau - \frac{\gamma}{\beta} \sin \beta|\tau|$). Before selecting one of these models it should be verified, that the oscillating behaviour is not a result of the estimation error (which depends on the sample size) or of the measurement error. The dependence of the estimations of correlation functions, especially of their oscillating behaviour on the size of the sample is illustrated in Figure 12. Here, the correlation function of the Zwickau-G profile and of three Monte-Carlo profiles together with the model correlation function $\sigma^2 e^{-\gamma|\tau|}$ are plotted. While the theoretical correlation function of the Monte-Carlo profiles - $\sigma^2 e^{-\gamma|\tau|}$ - is non-oscillating, the low size sample M1 shows a significant oscillating behaviour. These oscillations become smaller if the sample size increases and nearly vanish in the case of the very large sample M3.

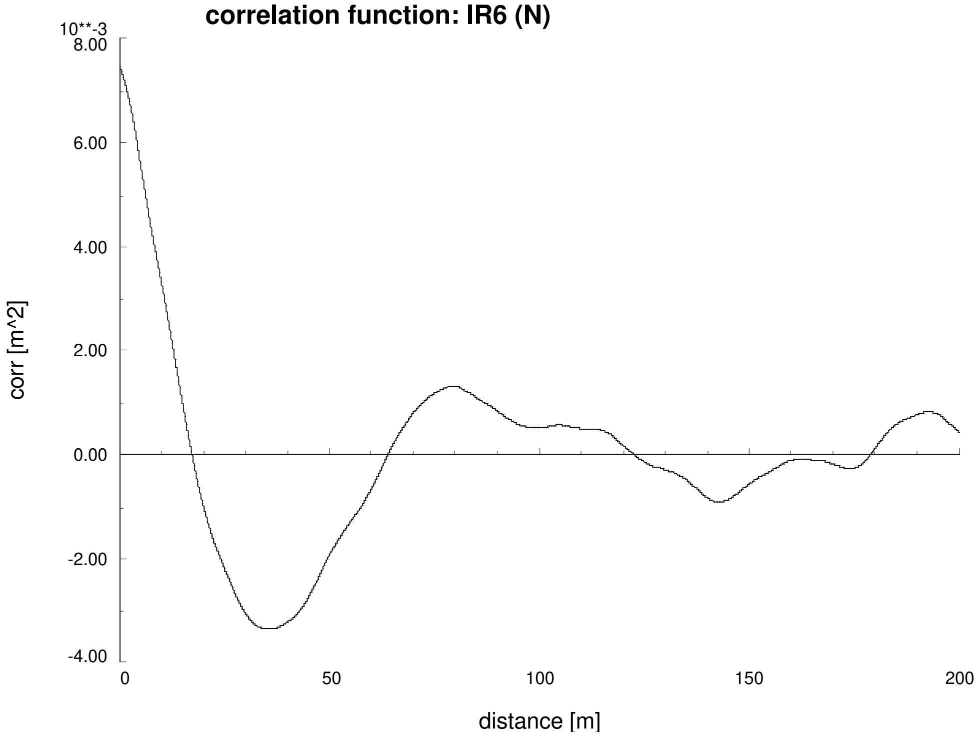

Figure 10: IR6-N profile - estimation of correlation function

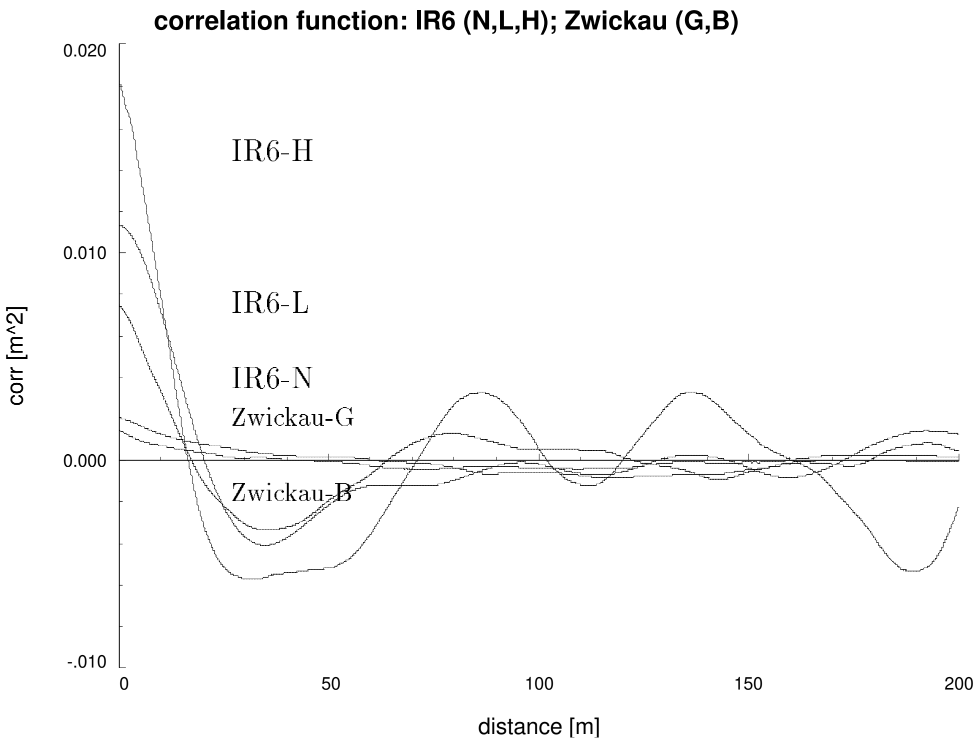

Figure 11: IR6 and Zwickau profiles - estimation of correlation functions

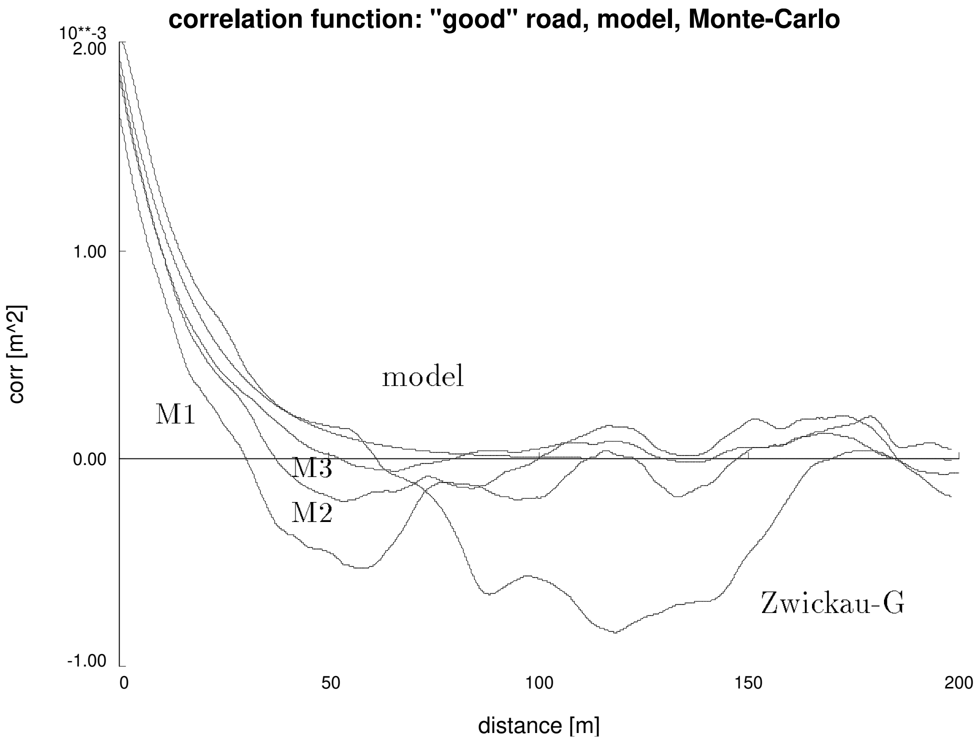

Figure 12: Zwickau-G, Monte-Carlo M1, M2, M3 profile - estimated and model correlation functions

Now spectral density functions are considered. Figure 13 shows graphs of three estimates of the spectral density function of IR6-N profile which are plotted versus the circular frequency $\alpha = \frac{2\pi}{L}$, where L denotes the corresponding wavelength. The highly fluctuating graph is the so-called periodogram of the sample calculated from the squares of the coefficients of the Fourier transformation of the sample.

To avoid these fluctuating behaviour, which is typical for estimates of spectral densities, periodogram smoothing procedures have been applied (see e.g. [3]). A weak smoothing is obtained calculating a weighted average of 13 neighbouring periodogram values for each frequency. The strong smoothing averages a considerably larger number of values (depending on square root of sample size).

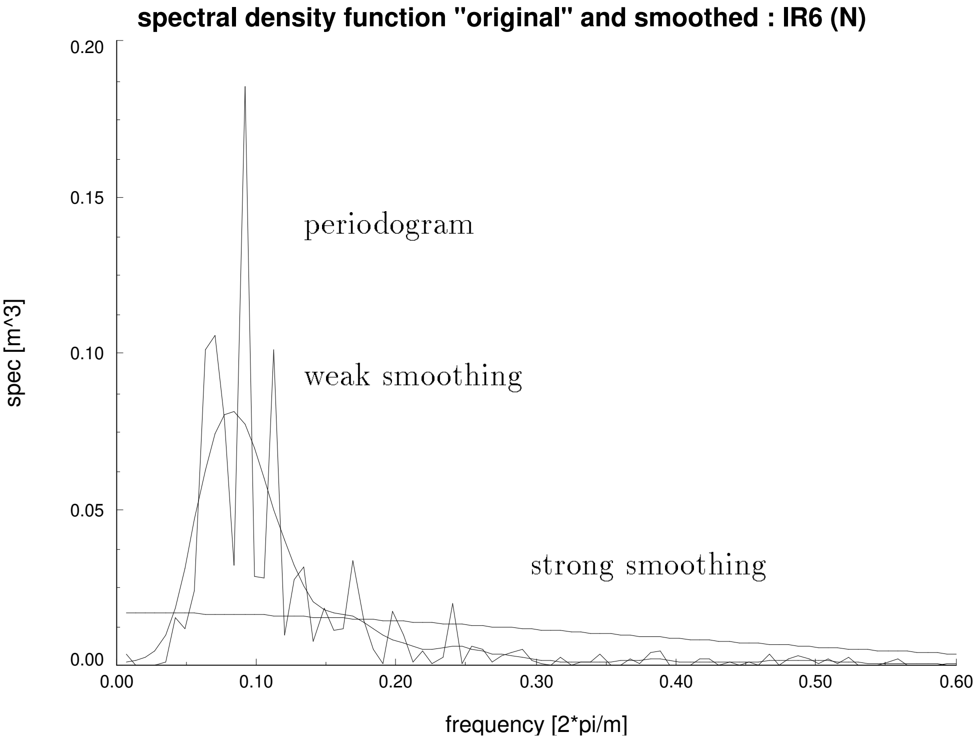

Figure 13: IR6-N profile - estimation of spectral density functions

Figure 14 compares the estimates (weak smoothing) of the spectral density functions of the IR6 profiles and Figure 15 compares the IR6-N profile with the Zwickau profiles. While the estimates can be calculated for values of the circular frequency up to $30 \ldots 60$ $[\frac{2\pi}{m}]$ (depending on the sampling frequency and the speed of the measuring vehicle) only the frequencies less than 0.3 seem to be significantly. They correspond to the long wave oscillations in the profiles mentioned above. The IR6 spectral density functions show maximum values at $\approx 0.08 \ldots 0.10$ $[\frac{2\pi}{m}]$ $(L = 60 \ldots 80\ [m])$ and the Zwickau profiles at ≈ 0.04 $[\frac{2\pi}{m}]$ $(L = 160\ [m])$.

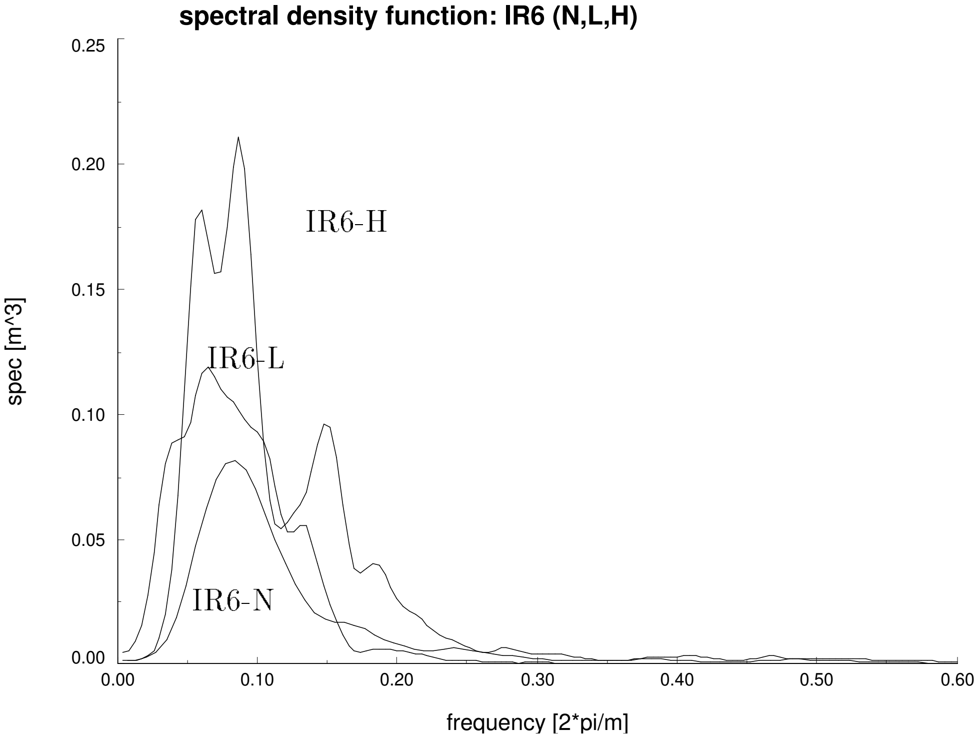

Figure 14: IR6 profiles - estimation of spectral density functions

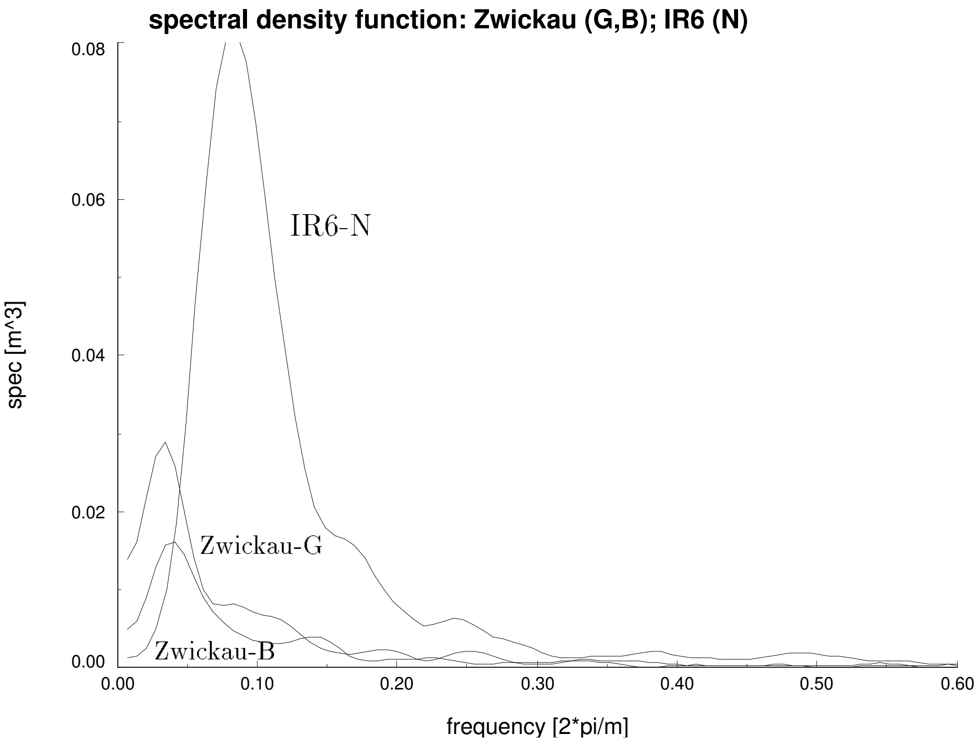

Figure 15: IR6-N and Zwickau profiles - estimation of spectral density functions

A widely used representation of spectral density functions of random road profiles uses logarithmic scaled axes (see Figure 16, taken from [1]). Since these "log-spectra" of road profile samples often show a linear behaviour over a certain frequency interval, the model function

$$S(\alpha) = S_0 \alpha^w$$

has been proposed, where $S_0 > 0$ and the parameter w is found to be $w \approx -2$. This model function cannot be extended to the whole frequency range but the model functions given in Table 3 show a similar behaviour for sufficiently large frequencies, since it holds $S(\alpha) = O(\alpha^{-2})$ for $\alpha \to \infty$ and the model parameters $(\sigma^2, \gamma, \beta)$ can be found by fitting the model to the data.

Figure 17 shows the estimate of the spectral density function (first and second smoothing) of the IR6-N profile in logarithmic scaled axes and Figure 18 compares the estimates (strong smoothing) of the IR6, Zwickau and Monte-Carlo M3 profiles. It can be observed that the weak smoothing fluctuates considerably for higher frequencies. Thatswhy the strong smoothings have been used for comparing graphs in Figure 18.

The local maximum of IR6-N spectrum at $\log \alpha \approx 1.4$ or $\alpha \approx 25$ indicates the short wave oscillations mentioned in the dicussion of Figure 7. While the Zwickau and the Monte-Carlo profiles show a good linear decrease with $w \approx -2$ in a frequency range $(10^{-0.5}, 10^1)$ the IR6 "log-spectra" show a steeper ($w \approx -3$) and less linear descent.

5 Conclusions

Three groups of random road profile samples have been investigated. Basic statistical characteristics as sample mean and standard deviation as well as (auto-)correlation and spectral density function have been estimated from the samples.

Correlation functions show a fast (exponential) decrease and an oscillating behaviour which seems to vanish if the sample size increases. Spectral density functions have (global) maximum values for very small frequencies related to long wave oscillations which dominate the profile plots in the IR6 group. For large frequencies the $O(\alpha^{-2})$ behaviour, especially in the case of the Zwickau profiles, can be observed.

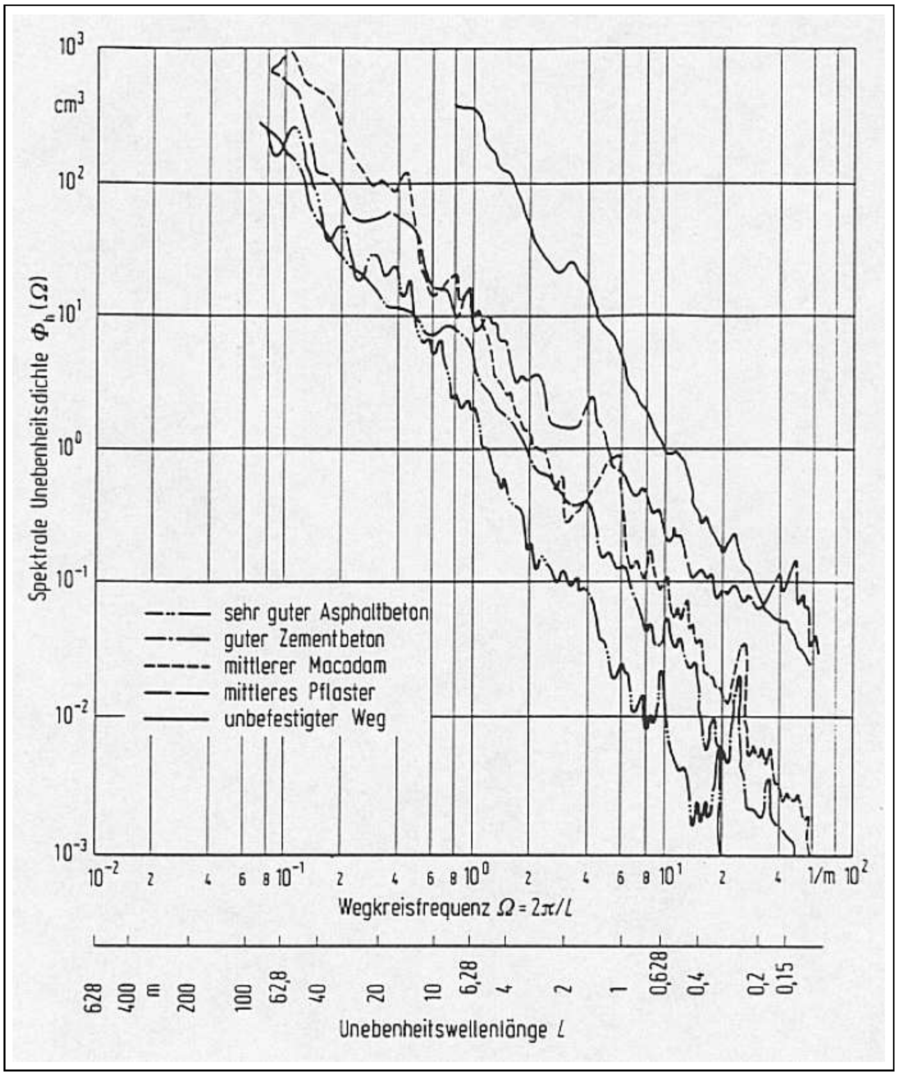

Figure 16: Log-Spectra of various road profile samples (taken from [1])

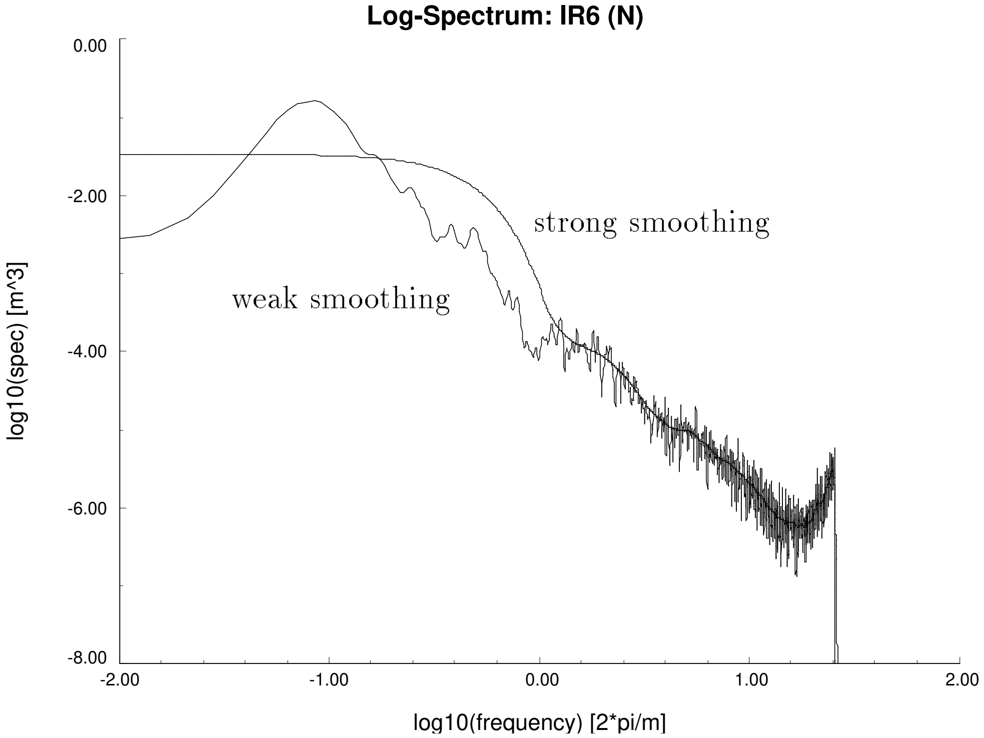

Figure 17: IR6-N profile - log-spectrum

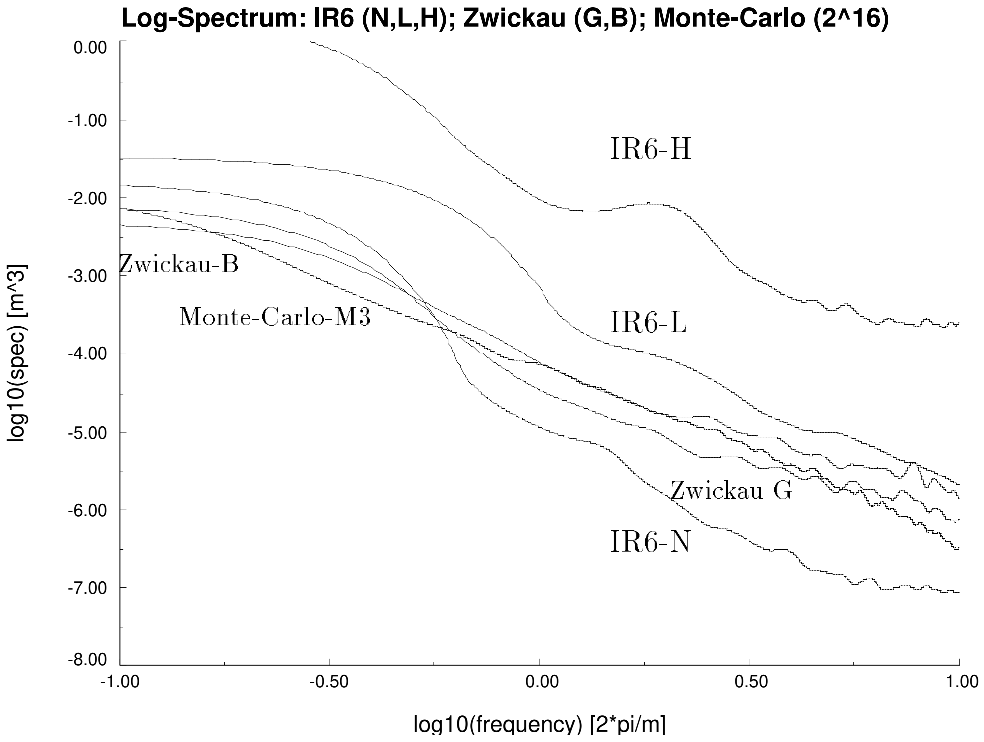

Figure 18: IR6, Zwickau and Monte-Carlo M3 profile - log-spectrum

References

[1] Mitschke, M.: Dynamik der Kraftfahrzeuge, Band B: Schwingungen.Springer, Berlin (1997)

[2] Popp, K. and Schiehlen, W.: Fahrzeugdynamik.Teubner, Stuttgart (1993)

[3] Priestley, M.B.: Spectral analysis and time series. Academic Press, London (1981)