

## Numerical Methods for ODEs

### Explicit Runge-Kutta-Methods

Number stages $r$	1	2	3	4–5	6	7–8	9–10	11	12–16	17
$2r + \frac{1}{2}r(r-1)$ Parameters	2	5	9	14–20	27	35–44	54–65	77	90–152	170
$r + 1$ conditions (3.24)	2	3	4	5–6	7	8–9	10–11	12	13–17	18
max. order $p$	1	2	3	4	5	6	7	8	9	10
order conditions $q$	0	1	3	7	16	36	84	199	485	1204

TABLE: Relation between number of stages, order, number of order conditions for explicit Runge-Kutta methods.

(a) **improved Euler-method** (order 2) (b) **Heun-2 method** (order 2)

$$\begin{array}{c|cc} 0 & & \\ 1/2 & 1/2 & \\ \hline & 0 & 1 \end{array}$$

$$\begin{array}{c|cc} 0 & & \\ 1 & 1 & \\ \hline & 1/2 & 1/2 \end{array}$$

(c) **Heun-3 method** (order 3)

(d) **Kutta method** (order 3)

$$\begin{array}{c|ccc} 0 & & & \\ 1/3 & 1/3 & & \\ 2/3 & 0 & 2/3 & \\ \hline & 1/4 & 0 & 3/4 \end{array}$$

$$\begin{array}{c|ccc} 0 & & & \\ 1/2 & 1/2 & & \\ 1 & -1 & 2 & \\ \hline & 1/6 & 2/3 & 1/6 \end{array}$$

(e) **Classical Runge-Kutta method** (order 4) (f) **3/8-rule** (order 4)

$$\begin{array}{c|cccc} 0 & & & & \\ 1/2 & 1/2 & & & \\ 1/2 & 0 & 1/2 & & \\ 1 & 0 & 0 & 1 & \\ \hline & 1/6 & 1/3 & 1/3 & 1/6 \end{array} .$$

$$\begin{array}{c|cccc} 0 & & & & \\ 1/3 & 1/3 & & & \\ 2/3 & -1/3 & 1 & & \\ 1 & 1 & -1 & 1 & \\ \hline & 1/8 & 3/8 & 3/8 & 1/8 \end{array} .$$