Technische Universität Chemnitz
Dr. M. Winkler Chemnitz, 15th January 2018

Numerical Methods for ODEs

Sheet 7

Exercise 22: Discontinuous Galerkin methods

Let $a \in \mathbb{R}$ and $f \in C(0,T;\mathbb{R})$ be given. We want to solve the initial value problem

$$\begin{cases} y'(t) = a\,y(t) + f(t) & \text{for } t \in (0,T], \\ y(0) = y_0 \end{cases} \tag{1}$$

with a discontinuous Galerkin method on an equidistant decomposition

$$\mathcal{T} := \{(t_k, t_{k+1}] \colon k = 0, \ldots, N-1\}$$

with $t_k := k\,h$. The extended variational formulation reads:

Find $y_h \in V_h$ such that

$$\sum_{k=0}^{N-1} \left[\int_{t_k}^{t_{k+1}} \big(y_h'(t) - a\,y_h(t) - f(t)\big) \cdot v_h(t)\,\mathrm{d}t + [y_h]_k \cdot v_k^+ \right] + (y_h(0) - y_0) \cdot v_0^- = 0,$$

for all $v_h \in W_h$.

(a) Use a DG(1)-approach for the trial and test functions V_h and W_h, respectively.

(b) Use piecewise linear and globally continuous functions CG(1) for V_h, and piecewise constant functions DG(0) for W_h. The integrals should be approximated by a trapezoidal rule. Do you already know the resulting method?

Remark: A Galerkin method with $V_h \neq W_h$ is called *Petrov-Galerkin method*, see also Remark 5.7.

Homework 14: Implementation of DG methods

Implement the DG(0) approach from Example 5.5 as well as the two methods from Exercise 22 for the solution of (1) in Matlab. Use the function interface

```
[t,y] = dg0(a, f, y0, time, N)
```

where `a`, `f` and `y0` are the input data of our problem, `time` is the time horizon of our ODE and `N` is the number of subintervalls $\{(t_k, t_{k+1})\}_{k=0}^{N-1}$ used for the DG method. The output parameters are the grid points t and the solution y in the grid-points.

Choose the input data $a = -3$, $f(t) = -10\,\pi\,\mathrm{e}^{-3t}\,\sin(10\,\pi\,t)$ and $y_0 = 1$ which lead to the exact solution $y(t) = \mathrm{e}^{-3t}\,\cos(10\,\pi\,t)$.

Compute the experimental convergence rate in the norm $\|\cdot\|_{\infty,h}$. For some methods you will probably observe a higher convergence rate than predicted by Theorem 5.6. This is due to the fact that the error function $y - y_h$ fulfills some superconvergence effects in the grid points.