Technische Universität Chemnitz
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Numerical Methods for ODEs

Sheet 6

Exercise 18: Stability of the explicit midpoint rule

Investigate the explicit midpoint rule

$$y_{k+2} = y_k + 2\,h\,f(t_{k+1}, y_{k+1})$$

for zero-stability and A-stability.

Exercise 19: Stability of the Adams-formulas

Discuss $A(\alpha)$-stability and stiff-stability for the Adams-Bashforth- and Adams-Moulton-formulas.

Exercise 20: Stability and consistency

Let ρ be the first associated polynomial of a linear and <u>consistent</u> r-step method. Is it possible that all roots μ_j, $j = 1, \dots, r$ of ρ satisfy $|\mu_j| < 1$? Explain your decision.

Exercise 21: Alternative construction of the Heun-Method

Interpret the Heun-method as a predictor-corrector-method (compare Definition 4.39), where the predictor is the explicit Euler method and the corrector with $M = 1$ is the Crank-Nicolson-Method (implicit trapezoidal rule).

Homework 12: Implementation of a BDF-method and the dependency on initial values

(a) Implement the BDF-method of degree 3

$$\tfrac{1}{6}\big(11\,y_{k+3} - 18\,y_{k+2} + 9\,y_{k+1} - 2\,y_k\big) = h\,f_{k+3}. \tag{1}$$

The function interface has the form

```
function y = BDF3(f,Df,time,y0,h,OSM)
```

`y0` is an $n \times m$-Matrix with $m \in \{1, 2, 3\}$. `OSM` is a pointer to a one-step method which shall be used to compute the missing initial values in case of $m < 3$. Use a Newton-method to realize the computation of y_{k+3}. To this end, we need a function handle `Df` to the Jacobian of f.

(b) Test your method with the initial value problem

$$\dot{y} = \lambda\, y, \quad y(0) = 1, \quad T = 8 \tag{2}$$

for the parameter $\lambda = -1$, initial values

- $y_0 = 1$
- $y_0 = [1, \exp(-h)]$
- $y_0 = [1, \exp(-h), \exp(-2\,h)]$

and fixed grid size $h = 0.05$. Use the classical Runge-Kutta method for `OSM` which has order 4. Draw the 3 computed solutions into one plot.

(c) Apply your method to the IVP (2) with parameter $\lambda = -10$ and grid sizes

$$h_k = 0.5^k, \quad k = 0, 1, \ldots, 14.$$

Use the explicit Euler method and the classical Runge-Kutta method for `OSM`. Estimate the convergence order of the BDF-method for both choices and create a table containing the absolute error of the method and the estimated convergence orders. Plot the error against the grid size. What do you observe and how can one explain the results?

Homework 13: Construction of a two-step method of order 4

Show: There exists only one two-step method which has consistency order 4. Determine the coefficients $(\alpha_0, \alpha_1, \alpha_2)$ and $(\beta_0, \beta_1, \beta_2)$. What is the name of this method? Plot also the region of A-stability for this method in Matlab, compare `plot_stability_region_msv.m`.

Hint: You can solve the equation system based on the order conditions from Theorem 4.11 in Matlab.