

Mathematische Methoden der Unsicherheitsquantifizierung

Sommersemester 2016

Overview of the `fem2d_lin`-MATLAB Routines

Introduction

The package `fem2d_lin` includes functions to solve and analyze elliptic boundary value problems of the form

$$-\nabla \cdot (a \nabla u) = f \text{ in } D, \quad u|_{\partial D} = g,$$

with $D = [0, L] \times [0, H] \subset \mathbb{R}^2$ using the Finite-Element Method (FEM). These finite elements are linear Lagrange elements on a uniform triangular grid.

Description of MATLAB Functions

- `fem2d_lin`:
This main function calculates the finite element solution of a given boundary value problem. The inputs for this function are length L and height H of the domain D , the coefficient function a and the source term f together with the boundary data g and the number of grid points. This routine returns coefficients vector of the solution w.r.t. the global FE basis as well as the global load vector and the global stiffness matrix together with the point and element table.
- `fem2d_lin_mesh`:
This sub function creates the point and element table for a given domain $D = [0, L] \times [0, H]$ and a given number of grid points in both directions. This function is used within the main routine `fem2d_lin`.
- `fem2d_lin_LHS`:
This sub function assembles the global stiffness matrix. The inputs are the point and element tables together with function values of a at given nodes. This function is used within the main routine `fem2d_lin`.
- `fem2d_lin_RHS`:
This sub function assembles the global load vector. The inputs are the point and element table together with function values of f at given nodes. This function is used within the main routine `fem2d_lin`.
- `fem2d_lin_plot`:
This function generates surface-contour plots of the FE solution (or every other function in the space spanned by FE basis functions = Galerkin space). The inputs are the function values of the solution u at grid points, the point and element table.
- `fem2d_lin_eval`:
This function calculates the function value of the FE solution (or every other function in the Galerkin space) at any point in the domain D . The inputs are the two coordinates of all points to be used for calculation, the vector of coefficients of the FE solution, point and element table. This function is particularly useful for an error estimation on a finer reference grid.
- `fem2d_lin_norm`:
This function computes the $L^2(D)$ and $H^1(D)$ norm of the FE solution (or every other function in the Galerkin space) exactly. Furthermore, this function can also output the global mass matrix of the FE basis functions and the global mass matrix for the gradients of the FE basis functions. The inputs are the vector of coefficients of the FE solution, point and element table. If the previously mentioned mass matrices are given as input the calculation time is reduced significantly.

- `fem2d_lin_demo`:

This script demonstrates the application of the `fem2d_lin`-package. Beside solving a specific variational problem and plotting the corresponding solution, it also tests for the convergence rates of the FE solution w.r.t. mesh width h measured in the $L^2(D)$ and $H^1(D)$ norm.

Exercise 1

Complete the function

```
fem2d_lin_int(u, ptable, eltable, Mint)
```

which shall compute the exact integral

$$\int_D u_h(x, y) \, d(x, y)$$

of a function u_h from the Galerkin space given its coefficient vector `u` as well as the point and element table `ptable` and `eltable`, resp., of the associated triangulation. (The fourth argument is optional, see code for description.)

Use the code of `fem2d_lin_norm` for orientation.