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Mathematische Methoden der Unsicherheitsquantifizierung

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Exercise Sheet 9: Stochastic Collocation

Exercise 1

We want to implement the full tensor collocation in MATLAB , i.e. we want to implement the quadrature operator

$$\mathcal{Q}_{\boldsymbol{p}} = \bigotimes_{i=1}^{d} \mathcal{Q}_{p_i}^{(i)}, \qquad \mathcal{Q}_{p_i}^{(i)} f = \sum_{k=1}^{1+p_i} w_k^{(i)} \, f\left(x_k^{(i)}\right)$$

where $\mathcal{Q}_{p_i}^{(i)}$ are the one dimensional Gauss quadrature operators with weights $w_k^{(i)}$ and points $x_k^{(i)}$. Develop a MATLAB script

```
[X,w] = FullGridSC(d, p, s)
```

where

- `d` is the number of random variables,
- `p` is a d dimensional vector in the natural numbers showing the degree of the polynomial in every dimension,
- `s` is a d dimensional cell array of strings containing either “legendre” or “hermite” and shows the quadrature points and weights in every dimension,
- `X` is the resulting grid of d dimensional collocation points and `w` the associated quadrature weights.

Hint: You can use the available scripts `legendre` and `hermite` which calculate the points and weights for Gauss quadrature with respect to a uniform and a standard normal distribution.

Exercise 2

We take a look at Exercise 5 of the second exercise sheet again:

Compute the average area A_μ of a random triangle which points are uniformly distributed inside the unit square with Monte Carlo simulation.

(a) Calculate A_μ by stochastic collocation or Gauss quadrature. Use polynomials of degree $p = 0, \ldots, 4$ for each of the six stochastic variables. Display the error of the collocation and quadrature method with respect to the number of collocation points $N_p = (1+p)^6$ in a log-log plot.

Hint: The true value is $A_\mu = 11/144$.

(b) Calculate A_μ with Monte Carlo simulation. Use $M = N_p = (1+p)^6$, $p = 0, \ldots, 4$ realizations and compute for each M 5 simulations. Display the error with respect to M in the same log-log plot as in Task (a).

(c) What do you observe? Explain your observations! What will change if we are interested in the mean of the squared area of the triangle?

Hint: This true value is $3/288$.

Exercise 3

We study the following random variation problem on $D = [0,1]^2$:

$$\int_D \nabla u(x,y,\boldsymbol{\xi}) \cdot \nabla v(x,y)\,\mathrm{d}(x,y) = \int_D f(x,y,\boldsymbol{\xi})\, v(x,y)\,\mathrm{d}(x,y) \quad \forall v \in H_0^1(D)$$
$$u(x,y,\boldsymbol{\xi})\big|_{\partial D} = g(x,y,\boldsymbol{\xi})\big|_{\partial D}$$

with

$$g(x,y,\boldsymbol{\xi}) = \sin(\xi_1 + x + y),$$
$$f(x,\boldsymbol{\xi}) = -\exp(\xi_2)\ (1 + \xi_3\ \sin(\pi x) + \xi_4\ \cos(\pi y))$$

and $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4)$ where $\xi_1 \sim U(0,1)$, $\xi_2, \xi_3, \xi_4 \sim N(0,1)$ are independent.

(a) Calculate the expectation $\mathbf{E}u(x)$ by a deterministic variation problem. Use the `fem2d_lin` scripts and a triangulation with $n = 101$ grid points in each direction.

(b) Calculate the expectation $\mathbf{E}u(x) \approx \mathbf{E}u_{\boldsymbol{p}}(x)$ by stochastic collocation. Use polynomials of degree $p = 0,\dots,4$ for each variable $\xi_1,\dots,\xi_4$. Display the behavior of the error $\|\mathbf{E}u(x) - \mathbf{E}u_p(x)\|_{H^1(D)}$ with respect to the number of collocation points $N_p = (1+p)^4$ in a log-log plot.

Hint: Use the result from (a) for the error calculations.

(c) Calculate the expectation $\mathbf{E}u(x) \approx \mathbf{E}u_M(x)$ by Monte Carlo simulation. Use $M = N_p = (1+p)^4$, $p = 0,\dots,4$ realizations. Compute for each M 5 simulations and display the behavior of the error $\|\mathbf{E}u(x) - \mathbf{E}u_M(x)\|_{H^1(D)}$ with respect to M in the same log-log plot as in Task (b).

Hint: Use again the result from (a) for the error calculations.