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Mathematische Methoden der Unsicherheitsquantifizierung

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Exercise Sheet 3: Monte Carlo Methods and Simulation Methods

Aufgabe 1

How would you simulate realizations from a Cauchy distribution $\mathrm{Cauchy}(0,1)$ with probability density function

$$f(x) = \frac{1}{\pi}\frac{1}{1+x^2}?$$

Use Monte Carlo simulation to calculate the expected value $\mathbf{E}[X]$ where $X \sim \mathrm{Cauchy}(0,1)$ with $M_j = 2^j \cdot 100$ realizations, $j = 0, \ldots, 10$. What do you observe?

Aufgabe 2

Develop an algorithm to simulate a uniform distribution on the unit simplex in $\mathbb{R}^n$ based on the rejection method.
How efficient is this algorithm with respect to n? In order to describe this, use the average number of trials until acceptance as a function of n.

Aufgabe 3

For a simulation of a uniform distribution on the unit sphere $\mathcal{S}^{n-1}$ in $\mathbb{R}^n$, the following statements are useful:

- If X is a n dimensional random vector with rotation invariant probability density function then $X/\|X\|$ is uniformly distributed on the unit sphere $\mathcal{S}^{n-1}$.
- If $X = (X_1, \ldots, X_3)$ is uniformly distributed on the unit sphere $\mathcal{S}^2$ in $\mathbb{R}^3$ then there holds $X_i \sim \mathrm{Uni}[-1,1]$ for $i = 1,2,3$.

For a proof see [1, Theorem 4.6 & Lemma 4.7]

(a) How can you simulate the uniform distribution on $\mathcal{S}^{n-1}$ using the rejection method?

(b) How can you simluate the uniform distribution on S^{n-1} directly?

(c) Someone suggests to simulate the uniform distribution on $\mathcal{S}^{n-1}$ using the random variable $S := X/\|X\|$ where

$$X = (2U_1 - 1, 2U_2 - 1, 2U_3 - 1), \quad U_1, U_2, U_3 \sim \mathrm{Uni}[0,1],$$

What do you think about this?

Implement all three algorithms and display your results in suitable plots.

Reference

[1] T. MÜLLER-GRONBACH, E. NOVAK, UND K. RITTER, *Monte Carlo Algorithmen*, Springer, Berlin Heidelberg, 2012. (E-Ressource of the TUC library).