Approximationstheorie

Vorlesung im Sommersemester 2018
Literaturliste

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1 Books

- Trefethen (2013) is the centerpiece of this class.
- Bornemann (2013). A very clear, insightful and humorous yet rigorous introduction to complex analysis (Funktionentheorie). In German.
- Wegert (2012) : Another introductory complex analysis text based on the Weierstraß perspective making extensive use of phase plots of complex functions, increasingly referred to as 'Wegert plots'.
- An in-depth treatment of error analysis for numerical algorithms is the monograph Higham (2002), which also includes an abstract definition of condition number and the rounding error counters employed in the stability analysis of the barycentric interpolation formula.

2 Chapter 2

The electrostatic analogy for Chebyshev points is used in Hesthaven (1998) to design quadrature rules for simplices.

3 Chapter 3

- The papers Austin et al. (2014) and Wright et al. (2015) explore the Laurent and Fourier settings which are deemphasized relative to the Chebyshev setting in ATAP. The former explores numerical algorithms based on approximating a function analytic on the unit disk based on function values at the roots of unity; the latter describes the extension of Chebfun to periodic functions.

- Shadrin (2004) gives a thorough overview of Markov's Theorem on the uniform operator norm of the derivative operator restricted to polynomials of finite degree and the maximality of the Chebyshev polynomials including interesting historical references.

4 Chapter 5

- The paper that re-popularized barycentric Lagrange interpolation is Berrut et al. (2004).
- A stability analysis of the barycentric interpolation formulas is given in Higham (2004).
- Webb et al. (2012) revisits the barycentric interpolation formulas to analyze their stability when evaluating the interpolation outside the unit interval (extrapolation). They find that the weaker stability properties of the second barycentric formula are exacerbated in this setting. The impressive numerical illustrations also recall the capability of rational interpolation to approximate meromorphic functions beyond the nearest singularity to $[-1, 1]$.

5 Chapter 6

- A nice presentation of the Weierstrass approximation theorem in $\mathbb{R}^n$ via the heat kernel along with the background on Fourier transforms and convolutions can be found in Folland (1995).
- Bernstein's proof of the Weierstrass approximation theorem is taken from Lorentz (1986).

6 Chapter 10

- Additional results on best approximation by polynomials as well as more details on the *Remez algorithm* can be found in the monographs of Phillips (2003, Section 2.4) and Powell (1981, Chapters 7,8).

7 Chapter 12

- Ransford (1995) gives a self-contained and rigorous account of potential theory in the complex plane.
- The application of complex potential theory to polynomial approximation is surveyed in Finkelshtein (2006), Levin et al. (2006) and Saff (2009).
- The monograph Fornberg (1996) on spectral methods provides a lucid and succinct exposition of the classical result on convergence of interpolation for analytic functions (Section 3.4, page 28 and Appendix E). It also gives a number of references to the literature of this topic, remarking that a simple formulation of the basic results is difficult to find.
- The classical treatise Krylov (1962) of V. I. Krylov on numerical quadrature (translated by Stroud) contains a detailed potential-theoretic analysis of the convergence of polynomial interpolation for analytic functions.

References

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