Feynman path integral regularization using Fourier Integral Operator ζ-functions

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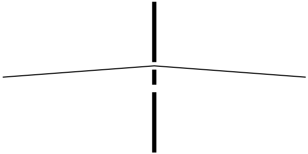
Feynman's alternative formulation of quantum mechanics

the double slit experiment



- interference pattern
- superposition principle
- probability density $P = |\Phi_1 + \Phi_2|^2$ (Φ_j quantum mechanical amplitudes)

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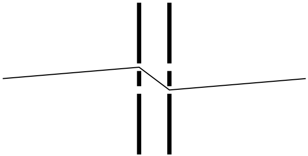
Feynman's alternative formulation of quantum mechanics

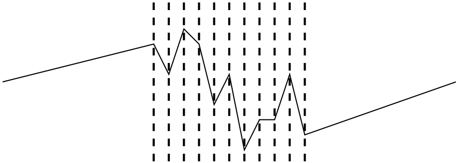
adding slits



- four possible paths
- quantum mechanical amplitudes Φ_j
- probability density $P = |\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4|^2$

Feynman's alternative formulation of quantum mechanics

more slits



probability density $P = \left|\sum_j \Phi_j\right|^2 = \left|\sum_{p\in\text{paths}} \Phi_p\right|^2$

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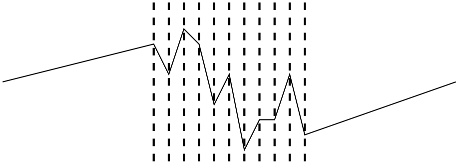

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Feynman

Let S_{cl} be the classical action. Then,

$$\forall p \in \text{paths}: \ \Phi_p = \exp\left(\frac{i}{\hbar} S_{\text{cl}}(p)\right)$$

Feynman's alternative formulation of quantum mechanics

- number of slits $\to \infty$
- size of / distance between slits $\to 0$
- continuous paths
- $\rightsquigarrow$ inductive limit

Feynman path integral

Propagator of a particle moving from (t_0, x_0) to (t_1, x_1):

$$K(t_1, x_1; t_0, x_0) = \int_{p \in \text{paths}((t_0,x_0)\to(t_1,x_1))} \exp\left(\frac{i}{\hbar} S_{\text{cl}}(p)\right) \mathcal{D}p$$

QuantumChromoDynamics

asymptotic freedom	confinement
length scales $\ll$ 1fm	length scales $\gtrsim$ 1fm
world of quarks and gluons	world of hadrons and glue balls
perturbatitive description	non-perturbative methods

Schrödinger equation and time evolution

- time evolution:

$$K(t,x;t',x') = \langle \delta_x, U(t,t')\delta'_x \rangle$$

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- Hamiltonian formulation (state vector ψ)

$$\psi(t) = U(t,t')\psi(t') = \exp\left(\frac{-i}{\hbar}\int_{t'}^{t} H(s)ds\right)\psi(t')$$

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- H is well-defined and can be constructed without computing K!

Partition function

Consider a time torus of length T. Then, (for the state vector ψ)

$$\psi(0) = \psi(T) = \exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)\psi(0).$$

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$$\psi(0) = \psi(T) = \exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)\psi(0).$$

$\Rightarrow$ partition function (statistical properties of thermal equilibrium)

$$\mathcal{Z}_T = \operatorname{tr}\exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)$$

Observables and their vacuum expectation values

Given an observable Ω, its toroidal vacuum expectation is given by

$$\begin{aligned}\langle\Omega\rangle_T :=& \frac{\operatorname{tr}\left(\exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)\Omega\right)}{\mathcal{Z}_T}\\ =& \frac{\operatorname{tr}\left(\exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)\Omega\right)}{\operatorname{tr}\exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)}.\end{aligned}$$

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Recover vacuum expectation of Ω using the thermal limit

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the operators $\exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)\Omega$

The operator $A := \exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)\Omega$ on X is an integral operator with kernel

$$k(x,y) = \int_{\mathbb{R}_{>0}}\int_{\partial B_{\mathbb{R}^N}} e^{i\langle x-y,r\xi\rangle} e^{ih_2(x,\xi)r^2+ih_1(x,\xi)r} a(x,r,\xi)d\xi dr.$$

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If X is a compact orientable C^∞-manifold, then the trace formally evaluates to

$$\begin{aligned}\mathrm{tr}A &= \int_X k(x,x)d\mathrm{vol}_X(x)\\ &= \int_X\int_{\mathbb{R}_{>0}}\int_{\partial B_{\mathbb{R}^N}} e^{ih_2(x,\xi)r^2+ih_1(x,\xi)r} a(x,r,\xi)d\xi dr d\mathrm{vol}_X(x)\end{aligned}$$

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gauging $\exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)\Omega$

Let $\mathfrak{g}$ be a holomorphic family such that

$$\mathfrak{g}(0) = 1 \quad \wedge \quad \mathfrak{g}(z)(x,r,\xi) = r^z \mathfrak{g}_\partial(z)(x,\xi).$$

Then, there exists $R \in \mathbb{R}$ such that for every $z \in \mathbb{C}_{\Re(\cdot)<R}$ the operator $A(z)$ with kernel

$$k(z)(x,y) = \int_{\mathbb{R}_{>0}} \int_{\partial B_{\mathbb{R}^N}} e^{i\langle x-y, r\xi\rangle} e^{ih_2 r^2 + ih_1 r} a\mathfrak{g}(z) d\xi dr.$$

is of trace class and satisfies

$$\begin{aligned}\mathrm{tr}A(z) &= \int_X k(z)(x,x) d\mathrm{vol}_X(x)\\ &= \int_X \int_{\mathbb{R}_{>0}} \int_{\partial B_{\mathbb{R}^N}} e^{ih_2 r^2 + ih_1 r} a\mathfrak{g}(z) d\xi dr d\mathrm{vol}_X(x).\end{aligned}$$

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- $\mathbb{C}_{\Re(\cdot)<R} \ni z \mapsto \mathrm{tr} A(z) \in \mathbb{C}$ has a holomorphic extension $\zeta(A)$

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- The fraction

$$\langle\Omega\rangle_T = \frac{\mathrm{tr}\left(\exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)\Omega\right)}{\mathrm{tr}\exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right)}$$

is almost always independent of the choice of gauge.

a new path integral

Definition

Let $\mathfrak{G}_T$ be a gauged family of operators with gauged kernels of the form

$$k(z)(x,y) = \int_{\mathbb{R}_{>0}} \int_{\partial B_{\mathbb{R}^N}} e^{i\langle x-y, r\xi\rangle} e^{ih_2 r^2 + ih_1 r} a\mathfrak{g}(z) d\xi dr$$

and

$$\mathfrak{G}_T(0) = \exp\left(\frac{-i}{\hbar}\int_0^T H(s)ds\right).$$

Then, we define

$$\langle\Omega\rangle := \lim_{T\to\infty} \frac{\zeta(\mathfrak{G}_T\Omega)}{\zeta(\mathfrak{G}_T)}(0).$$

Free relativistic Fermion (in one spatial dimension)

Hamiltonian:

$$H = \begin{pmatrix} mc^2 & -i\hbar\partial \\ -i\hbar\partial & mc^2 \end{pmatrix} \sim \begin{pmatrix} mc^2 & \hbar r\xi \\ \hbar r\xi & mc^2 \end{pmatrix}$$

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Observable H (ground state energy)

$$\langle H\rangle = \lim_{T\to\infty}\lim_{z\to 0}\frac{\frac{1}{2\pi}\int_{\mathbb{R}}\int_{\mathbb{R}}\chi(x)\operatorname{tr}\left(\exp\left(\frac{-i}{\hbar}TH\right)H\mathfrak{g}(z)\right)d(r\xi)dx}{\frac{1}{2\pi}\int_{\mathbb{R}}\int_{\mathbb{R}}\chi(x)\operatorname{tr}\left(\exp\left(\frac{-i}{\hbar}TH\right)\mathfrak{g}(z)\right)d(r\xi)dx}$$

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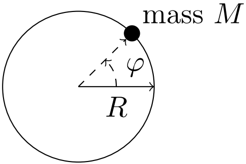
Topological rotor



- $J := MR^2$
- generalized momentum: $p = J\partial_0\varphi$
- Hamiltonian: $H = \frac{p^2}{2J}$
- topological charge: $Q = \frac{1}{2\pi}\int_0^T \frac{p}{J}$
- topological susceptibility:
  $\chi_{\text{top}} = \lim_{T\to\infty} \frac{\langle Q^2\rangle_T}{-iT}$

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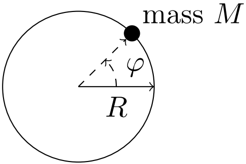

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Energy gap:

$$\Delta E = 2\pi^2\chi_{\text{top}}$$

Spontaneous Symmetry Breaking

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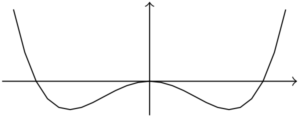

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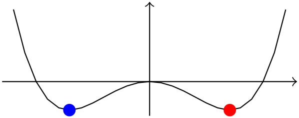

- ground state localizations are not symmetric

Spontaneous Symmetry Breaking

- scalar fields $\varphi^1, \ldots, \varphi^k$
- Hamiltonian: $H(\varphi^1, \ldots, \varphi^k)$
- partition function: $\mathcal{Z}_T(\varphi) = \zeta\left(\exp\left(\frac{-i}{\hbar}\int_0^T H(\varphi)(t)dt\right)\mathfrak{g}\right)(0)$

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$$V_e^T = \frac{\mathcal{Z}_T(\varphi)}{-iT\mathrm{vol}(X)} = \frac{\zeta\left(\exp\left(\frac{-i}{\hbar}\int_0^T H(\varphi)(t)dt\right)\mathfrak{g}\right)(0)}{-iT\mathrm{vol}(X)}$$

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- φ_0^j are vacuum expectation values of the φ^j (in the limit $T \to \infty$)
- $(\partial_i\partial_j V_e^T(\varphi_0))_{i,j}$ self-adjoint $\rightsquigarrow$ eigenvalues are squared field masses (in the limit $T \to \infty$)

Spontaneous Symmetry Breaking - the φ^4 model

- Hamiltonian: $H = \int_X \frac{p^2}{2} - \frac{\varphi\Delta\varphi}{2} - \frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4!}\varphi^4 dx$
- vacuum expectation values: $\varphi_0 = \pm\sqrt{\frac{6}{\lambda}}\mu$
- field mass: $\sqrt{2}\mu$

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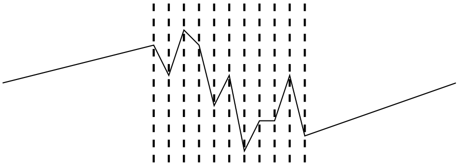
gauge dependence of $\mathcal{Z}_T$

The partition function

$$\mathcal{Z}_T(\varphi) = \zeta\left(\exp\left(\frac{-i}{\hbar}\int_0^T H(\varphi)(t)dt\right)\mathfrak{g}\right)(0),$$

and thus V_e^T, is independent of the particular choice of gauge $\mathfrak{g}$.

Lattice QFT



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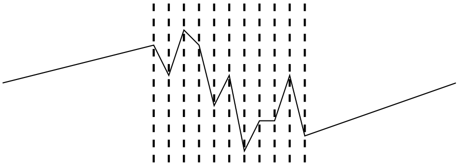

$$\mathcal{Z}_T^{\Omega} = \int_{\mathbb{R}^n} \exp\left(\frac{i}{\hbar} \int \text{Lagrangian}(p)\ d\text{vol}_{\text{universe}}\right) a(p) dp$$

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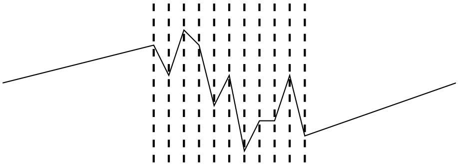

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Lattice QFT: Wick Rotation

$$\mathcal{Z}_T^{\Omega,\text{Wick}} = \int_{\mathbb{R}^n} \exp\left(-\frac{1}{\hbar}\int \text{Lagrangian}(p)\ d\text{vol}_{\text{universe}}\right) a(p)dp$$

"ζ-Lattice QFT"

(i) write down your LQFT

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$$\mathcal{Z}_T^\Omega(z) = \int_{\mathbb{R}^n} \exp\left(\frac{i}{\hbar}\int \text{Lagrangian}(p)\ d\text{vol}_{\text{universe}}\right) a(z)(p)dp$$

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(iii) construct distributionally equivalent family of Fourier Integral Operator "traces"

$$\mathcal{Z}_T^\Omega(z) = \sum_{\iota\in I} \int_{\partial B_{\mathbb{R}^n}} \int_{\mathbb{R}_{>0}} e^{ir\vartheta(\xi)} r^{d_\iota + z} \alpha_\iota(z)(\xi)\ dr\ d\xi$$

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(v) compute Laplace transform

$$\int_{\mathbb{R}_{>0}} e^{ir\vartheta(\xi)} r^{d_\iota + z} dr = \frac{\Gamma(d_\iota + z + 1)}{(-i\vartheta(\xi))^{d_\iota + z + 1}}$$

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(vi) and enjoy the remaining (fully regular) integrals

$$\mathcal{Z}_T^\Omega(0) = \sum_{\iota \in I} \Gamma(d_\iota + 1) \int_{\partial B_{\mathbb{R}^n}} \frac{\alpha_\iota(0)(\xi)}{(-i\vartheta(\xi))^{d_\iota + 1}} d\xi$$

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  $\rightsquigarrow$ needs regularization

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