MARINUS A. KAASHOEK:

half a century of operator theory

in

Amsterdam

Opening Lecture IWOTA 2017 (Chemnitz)

by

Harm Bart
Erasmus University Rotterdam

Born 1937

Ridderkerk

Studied in Leiden

Oldest University in The Netherlands

Founded in 1575

PhD in 1964 (Leiden)

Postdoc University of California at Los Angeles, 1965 - 1966

VU University Amsterdam 1966 – 2002

Emeritus Professor 2002 – $\cdots$ **(active!)**

Hence the title:

half a century of operator theory

in

Amsterdam

Many Ph.D students (**17**)

MatScinet: **234** publications

Co-author of **9** books

Lots of collaborators

Research interests mentioned in CV:

Analysis and Operator Theory,
and various connections between Operator Theory Matrix Theory and Mathematical Systems Theory

In particular, Wiener-Hopf integral equations and Toeplitz operators and their nonstationary variants

State space methods for problems in Analysis

Metric constrained interpolation problems,
and various extension and completion problems for partially defined matrices or operators,
including relaxed commutant lifting problems.

In the available time impossible to cover

all aspects

all connections

all references

Aim:
Just to give an impression on what Kaashoek has been working on

Emphasis on **ideas** / less on specific results

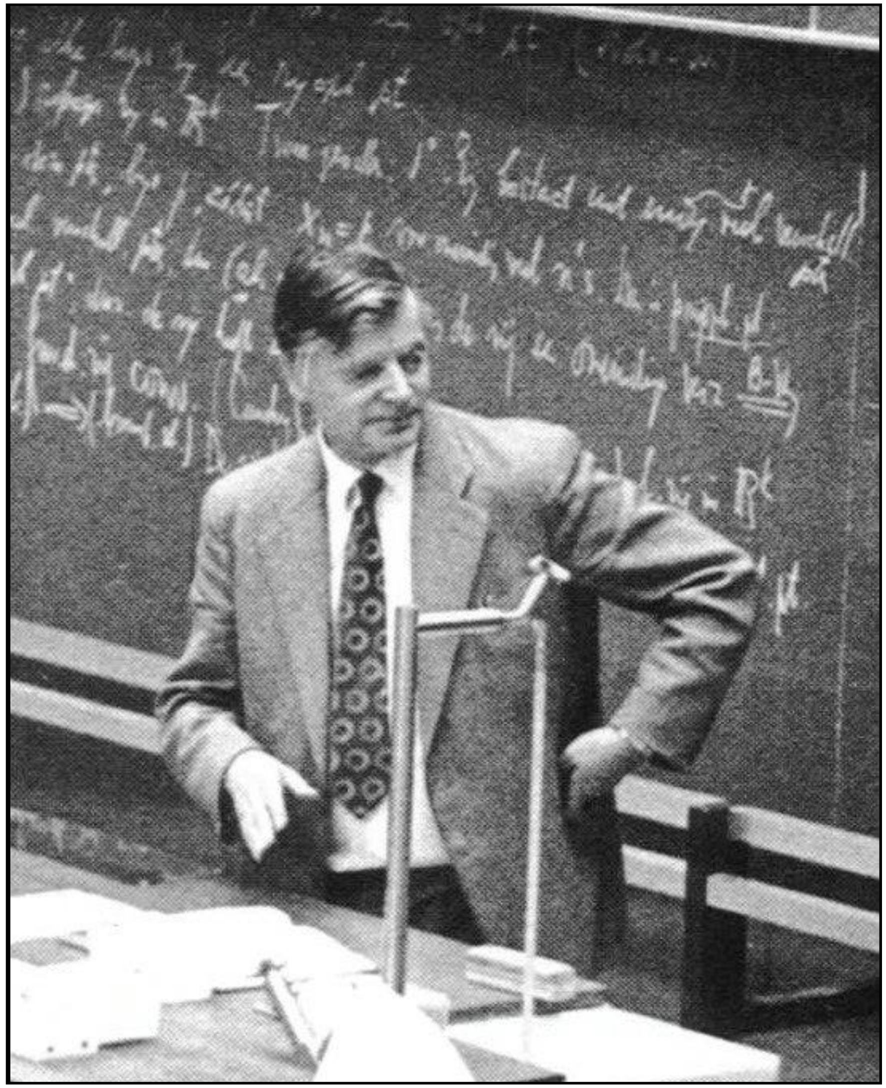
Not all the time mentioning of co-authors involved
List of them (MathSciNet):

Co-authors (by number of collaborations)

Alpay, Daniel   Arov, Damir Zyamovich   Ball, Joseph A.   Bart, Harm   Ben-Artzi, Asher   Brualdi, Richard A.   Böttcher, Albrecht   Dijksma, Aad   Dritschel, Michael A.   Dym, Harry   Foias, Ciprian   Frazho, Arthur E.   Fritzsche, Bernd   Förster, Karl-Heinz   Gohberg, Israel   Goldberg, Seymour   Groenewald, Gilbert J.   Haimovici, Iulian   Huijsmans, Charles Boudewijn   Iftime, Orest V.   Kim, Jeongook   Kirstein, Bernd   Kos, Johan   Lancaster, Peter   Langer, Heinz   Lay, David C.   Lerer, Leonid   Luxemburg, Wilhelmus Anthonius Josephus   Margulis, Igor   Markus, Alexander S.   Pik, Derk   Ran, André C. M.   Rodman, Leiba   Rovnyak, James   Sakhnovich, Alexander L.   Sasane, Amol J.   Schermer, J. N. M.   Smyth, M. R. F.   Spitkovsky, Ilya M.   Van Dooren, Paul M.   Verduyn Lunel, Sjoerd M.   West, Timothy Trevor   Woerdeman, Hugo J.   Zeinstra, Chris   de Pagter, Ben   ter Horst, Sanne   van Schagen, Frederik   van de Ven, M. P. A.   van der Mee, Cornelis V. M.

PhD in 1964

Supervisor A.C. Zaanen



Doctoral Thesis:

Closed linear operators on Banach spaces

One of the issues:

Local behavior of operator pencils $\lambda S - T$

Sufficient conditions for

$$\dim \operatorname{Ker}(\lambda S - T), \qquad \operatorname{codim} \operatorname{Im}(\lambda S - T)$$

to be constant on deleted neighborhood of the origin

Determination size of the neighborhood

Extension work of Gohberg/Krein and Kato

Postdoc University of California at Los Angeles, 1965 - 1966

Upon return to Amsterdam:

Suggestion to HB: try generalization

pencils $\lambda S - T \rightarrow$ **analytic operator functions** $W(\lambda)$

(admitting local power series expansions)

Important tool: linearization / reduction to the pencil case:

local properties $W(\lambda) \leftrightarrow$ local properties $\lambda\mathbf{S}_W - \mathbf{T}_W$

Suitable operators $\mathbf{S}_W$ and $\mathbf{T}_W$ on 'big(ger)' spaces

Defined in terms of power series expansion

$$W(\lambda) = \sum_{k=0}^{\infty} \lambda^k W_k$$

Inspired by (among others, but mainly)

Karl-Heinz Foerster († 29–1–2017)

Local properties $W(\lambda) \leftrightarrow$ **local** properties **pencil** $\lambda\mathbf{S}_W - \mathbf{T}_W$

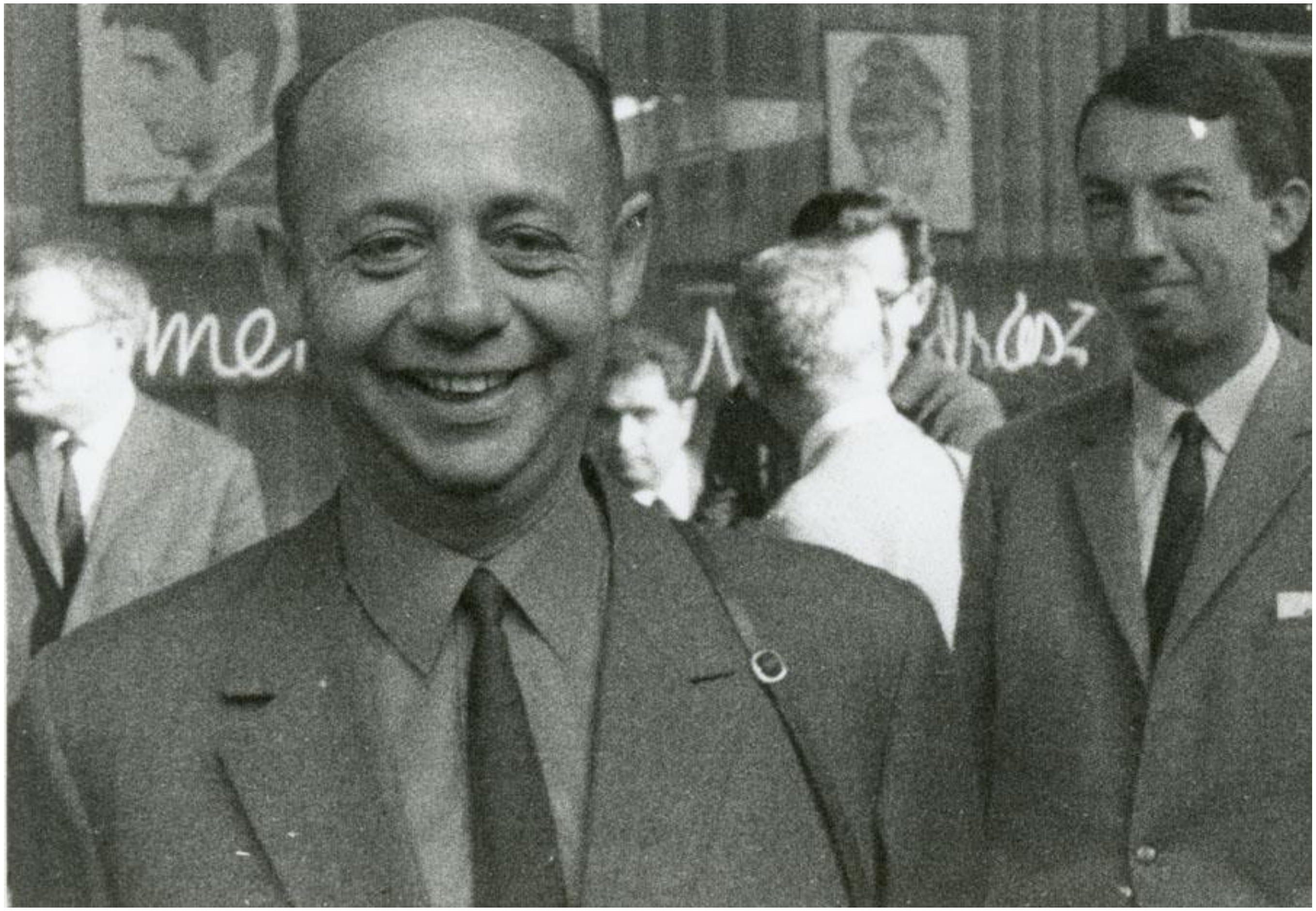
Drawbacks:

local behavior instead of **global** behavior

pencil $\lambda\mathbf{S}_W - \mathbf{T}_W$ instead of **spectral** item $\lambda\mathbf{I}_W - \mathbf{T}_W$

Helpful in some circumstances: $\mathbf{S}_W$ left invertible

1975: Enters Israel Gohberg



Gohberg, Kaashoek, Lay:

Global reduction to **spectral** case $\lambda\mathbf{I} - \mathbf{T}$

(*killing two birds with one stone*)

Linearization by **equivalence** after **extension**

$$\begin{bmatrix} W(\lambda) & 0 \\ 0 & I_Z \end{bmatrix} = E(\lambda)(\lambda\mathbf{I} - \mathbf{T}_W)F(\lambda)$$

$E(\lambda), F(\lambda)$ analytic equivalence functions

(Many) properties $W(\lambda) \leftrightarrow$ properties spectral pencil $\lambda\mathbf{I} - \mathbf{T}_W$

Further analysis

Underlying concept: **realization**

Representation in the form

$$W(\lambda) = D + C(\lambda I_X - A)^{-1}B \; (: Y \to Y)$$

Important case: $D = I_Y$

NB Misprint on next slide:

$$\widehat{y}(\lambda) = (D + C(\lambda - A)^{-1}B$$

should be

$$\widehat{y}(\lambda) = (D + C(\lambda - A)^{-1}B)\widehat{u}(\lambda)$$

Background (1)

Systems theory

Transfer function of linear time invariant system

$$\begin{cases} x'(t) = Ax(t) + Bu(t), & t \geq 0 \\ y(t) \; = \; Cx(t) + Du(t), & t \geq 0 \\ x(0) = 0 \end{cases}$$

Laplace transform:

$$\hat{y}(\lambda) = (D + C(\lambda - A)^{-1}B$$

Background (2): Livsic-Brodskii characteristic function

Characteristic functions of Livsic-Brodskii type, i.e.,

$$I_H + 2iK^*(\lambda I_G - A)^{-1}K, \qquad KK^* = \frac{1}{2i}(A - A*)$$

H, G Hilbert spaces

Designed to handle operators not far from being selfadjoint

Invariant subspace problem

Echo:
Bart, Gohberg, Kaashoek: Operator polynomials as inverses of characteristic functions, 1978
First paper in first issue of the newly founded IEOT

Realization takes different concrete forms depending on analyticity/continuity properties $W(\lambda)$

For instance:

- $W(\lambda)$ analytic on bounded Cauchy domain and continuous toward its boundary Γ (D identity operator)

- $W\lambda)$ analytic on bounded open set, no boundary requirement (Mitiagin, 1978)

- $W\lambda)$ rational matrix function, analytic at infinity (Systems Theory)

Connection with realization

$$W(\lambda) = D + C(\lambda I_X - A)^{-1}B$$

Linearization by equivalence after **two-sided** extension:

$$\begin{bmatrix} W(\lambda) & 0 \\ 0 & I_X \end{bmatrix} = E(\lambda) \begin{bmatrix} \lambda I_X - (A - BD^{-1}C) & 0 \\ 0 & I_Y \end{bmatrix} F(\lambda)$$

(Many) properties $W(\lambda) \leftrightarrow$ properties spectral pencil $\lambda I_X - A^{\times}$

$$A^{\times} = A\text{-}BD^{-1}C$$

$$W(\lambda)^{-1} = D^{-1} - D^{-1}C(\lambda I_X - A^{\times})^{-1}BD^{-1}$$

Realization, factorization, invariant subspaces

$$W(\lambda) = I_Y + C(\lambda I_X - A)^{-1}B$$

$$W(\lambda)^{-1} = I_Y \text{-} C(\lambda I_X - A^{\times})^{-1}B$$

($D = I_Y$ for simplicity)

M invariant subspace A

$M^{\times}$ invariant subspace $A^{\times} = A - BC$

Matching: $X = M \dot{+} M^{\times}$

Induces **factorization** $W(\lambda) = W_1(\lambda)W_2(\lambda)$

$$W_1(\lambda) = I_Y + C(\lambda I_X - A)^{-1}(I - P)B$$
$$W_2(\lambda) = I_Y + CP(\lambda I_X - A)^{-1}B$$

$P =$ **projection** of $X = M \dot{+} M^\times$ onto $M^\times$ along M

$$W_1(\lambda)^{-1} = I_Y - C(I - P)(\lambda I_X - A)^{-1}B$$
$$W_2(\lambda)^{-1} = I_Y - C(\lambda I_X - A)^{-1}PB$$

Factorization Principle

Bart/Gohberg/Kaashoek and **Van Dooren** **(1978)**

Opportunities:

- Choice realization $W(\lambda) = D + C(\lambda I_X - A)^{-1}B$

  for instance **minimal**

- Choice (matching) invariant subspaces M and $M^{\times}$

  for instance **spectral** subspaces

Corresponds to factorizations with special properties pertinent to the particular application at hand

- **Stability** of factorizations $\leftrightarrow$ **stability** invariant subspaces

Example:

The (vector-valued) **Wiener-Hopf integral equation**

$$\phi(t) - \int_0^\infty k(t-s)\phi(s)\,ds = f(t), \qquad t \geq 0$$

Kernel function $k \in L_1^{n\times n}(-\infty,\infty)$

Given function $f \in L_1^n[0,\infty)$

Desired solution function $\phi \in L_1^n[0,\infty)$

Associated **operator** $H : L_1^n[0,\infty) \to L_1^n[0,\infty)$

$$(H\phi)(t) = \phi(t) - \int_0^\infty k(t-s)\phi(s)\,ds, \qquad t \geq 0$$

Symbol: $W(\lambda) = I_n - \int_{-\infty}^{+\infty} e^{i\lambda t} k(t) dt$

Continuous on the real line

$\lim_{\lambda\in\mathbb{R},\,|\lambda|\to\infty} W(\lambda) = I_n$ (Riemann-Lebesgue)

Fredholm properties $H \leftrightarrow$ **factorization properties** $W(\lambda)$

$H : L_1^n[0, \infty) \to L_1^n[0, \infty)$ invertible

$\Updownarrow$

$W(\lambda)$ admits **canonical Wiener-Hopf factorization**

$$W(\lambda) = W_-(\lambda)W_+(\lambda)$$

Factors $W_-(\lambda)$ and $W_+(\lambda)$ satisfying certain analyticity, continuity and invertibility conditions on lower and upper half plane, respectively

Needed for effective description inverse H:

concrete knowledge $W_-(\lambda)$ and $W_+(\lambda)$

Application 'state space method' involving the use of realization

Assumption: $W(\lambda)$ rational $n \times n$ matrix function

Realization $W(\lambda) = I_n + C(\lambda I_m - A)^{-1}B$

A no real eigenvalue (continuity on the real line)

(Real line splits the **non-connected** spectrum of the $m \times m$ matrix A)

Application Factorization Principle:

$H : L_1^n[0,\infty) \to L_1^n[0,\infty)$ invertible

$$\Updownarrow$$

$A^\times = A - BC$ no real eigenvalue and $\mathbb{C}^m = M \dot{+} M^\times$

- M= spectral subspace A / upper half plane
- $M^\times$= spectral subspace $A^\times$ / lower half plane

Description inverse of H:

$$(H^{-1}f)(t) = f(t) + \int_0^\infty \kappa(t,s) f(s)\, ds, \qquad t \geq 0$$

$$\kappa(t,s) = \begin{cases} +iC\mathbf{e}^{-\mathbf{it}\mathbf{A}^\times} P \mathbf{e}^{\mathbf{is}\mathbf{A}^\times} B, & s < t, \\ -iC\mathbf{e}^{-\mathbf{it}\mathbf{A}^\times} (I_m - P) \mathbf{e}^{\mathbf{is}\mathbf{A}^\times} B, & s > t. \end{cases}$$

$P =$ **projection** of $\mathbb{C}^m = M \dot{+} M^\times$ onto $M^\times$ along M

NB: semigroups entering the picture!

Non-invertible case

Realization $W(\lambda) = I_n + C(\lambda I_m - A)^{-1}B$

A no real eigenvalue

Wiener-Hopf operator H Fredholm $\Leftrightarrow$ $A^\times$ no real eigenvalue

Fredholm characteristics:

$$\text{dim Ker}\, H = \text{dim}\,(M \cap M^\times)$$

$$\text{codim Im}\, H = \text{codim}\,(M + M^\times)$$

$$\text{index}\, H = \text{dim Ker}\, H - \text{codim Im}\, H = \text{dim}\, M - \text{codim} M^\times$$

Situation when $W(\lambda) = I_n + C(\lambda I_m - A)^{-1}B$ does **not** allow for a canonical Wiener-Hopf factorization

Non-canonical factorization:

$$W(\lambda) = W_-(\lambda) \begin{bmatrix} \left(\frac{\lambda-i}{\lambda+i}\right)^{\kappa_1} & 0 & \cdots & 0 \\ 0 & \left(\frac{\lambda-i}{\lambda+i}\right)^{\kappa_2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \left(\frac{\lambda-i}{\lambda+i}\right)^{\kappa_n} \end{bmatrix} W_+(\lambda)$$

$\kappa_1 \leq \kappa_2 \leq \cdots \leq \kappa_n$: **factorization indices (unique)**

Can be described **explicitly** in terms of A, B, C and $M, M^\times$

Similar approach works for

- Block Toeplitz operators
- Singular integral equations
- Riemann-Hilbert boundary value problem

Impression of additional applications

Titles Part IV-VII from the monograph
A State Space Approach to Canonical Factorization with Applications
(Bart, Gohberg, Kaashoek, Ran, OT 200, 2010):

- Factorization of selfadjoint matrix functions
- Riccati equations and factorization
- Factorization with symmetries

(Etc.)

Also in the monograph: application tot the **transport equation**

Integro-differential equation modeling radiative transfer in stellar atmosphere

Can be written as Wiener-Hopf integral equation with operator valued kernel

Employs **infinite dimensional version** of the Factorization Principle

Invertibility of the associated operator involves **matching** of two specific spectral subspaces of two concrete self-adjoint operators (albeit w.r.t. different inner products)

Transport equation: instance of **non-rational** case

Leads up to considering situations where there is no analyticity at infinity

Realizations $W(\lambda) = I_Y + C(\lambda I_X - A)^{-1}B$ involving **unbounded operators**

Considerable technical difficulties

Direct sum decompositions associated with **connected** spectra.

Led to new concept in semigroup theory: **bisemigroup**

S direct sum of of two possibly **unbounded** closed operators S_- and S_+

$-S_-$ and $+S_+$ generators of **exponentially decaying** semigroups

Bisemigroup generated by S:

$$E(t;S) = \begin{cases} -e^{tS_-}, & t < 0 \\ +e^{tS_+}, & t > 0 \end{cases}$$

Generalization of semigroup

Not to be confused with the notion of a **group**

Simple Example:

$$A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \qquad S = -iA = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Group generated by S:

$$e^{tS} = \begin{bmatrix} e^{t} & 0 \\ 0 & e^{-t} \end{bmatrix}, \qquad -\infty < t < +\infty$$

Bisemigroup generated by S:

$$E(t;S) = \begin{bmatrix} -e^{t} & 0 \\ 0 & 0 \end{bmatrix}, \qquad -\infty < t < 0$$

$$E(t;S) = \begin{bmatrix} 0 & 0 \\ 0 & e^{-t} \end{bmatrix}, \qquad 0 < t < +\infty$$

Bisemigroups introduced in BGK paper in
Journal of Functional Analysis 1986

Sparked off considerable 'follow up':

Cornelis van der Mee (student **Kaashoek**):
Exponentially dichotomous operators and applications
OT 182, 2008
Applications: Wiener-Hopf factorization and Riccati equations, transport equations, diffusion equations of indefinite Sturm-Liouville type, noncausal infinite dimensional linear continuous-time systems, and functional differential equations of mixed type

Semi-Plenary Talk IWOTA 2014
Christian Wyss: Dichotomy, spectral subspaces and unbounded projections

Umbrella:

State space method in analysis

OT 200: *A State Space Approach to Canonical factorization with Applications* (BGKR, 2010)

Still very much alive

Latest paper showing this in the title:

Frazho, Ter Horst, **Kaashoek**: **State space formulas** for a suboptimal rational Leech problem I: Maximum entropy solution (2014)

As indicated earlier: also other 'Kaashoek topics'

Mention here:

Completion, extension and interpolation problems

General issue:

Object **partly** known/given

Determine **missing** parts such that certain conditions are satisfied

Example: Positive completions of band matrices

$$\begin{bmatrix}
* & * & * & * & * & * & * & * & ? & ? & ? & ? & ? \\
* & * & * & * & * & * & * & * & * & ? & ? & ? & ? \\
* & * & * & * & * & * & * & * & * & * & ? & ? & ? \\
* & * & * & * & * & * & * & * & * & * & * & ? & ? \\
* & * & * & * & * & * & * & * & * & * & * & * & ? \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
? & * & * & * & * & * & * & * & * & * & * & * & * \\
? & ? & * & * & * & * & * & * & * & * & * & * & * \\
? & ? & ? & * & * & * & * & * & * & * & * & * & * \\
? & ? & ? & ? & * & * & * & * & * & * & * & * & * \\
? & ? & ? & ? & ? & * & * & * & * & * & * & * & *
\end{bmatrix}$$

Example: Strictly contractive completions

$$\begin{bmatrix} * & * & * & * & ? & ? & ? & ? \\ * & * & * & * & * & ? & ? & ? \\ * & * & * & * & * & * & ? & ? \\ * & * & * & * & * & * & * & ? \\ * & * & * & * & * & * & * & * \end{bmatrix} \qquad (1)$$

Reduction to positive extension problem for band matrix:

$$\begin{bmatrix} I_3 & (1) \\ (1)^* & I_8 \end{bmatrix}$$

$$\|$$

Positive extension band matrix

$$\begin{bmatrix}
\mathbf{1} & 0 & 0 & 0 & 0 & * & * & * & * & ? & ? & ? & ? \\
0 & \mathbf{1} & 0 & 0 & 0 & * & * & * & * & * & ? & ? & ? \\
0 & 0 & \mathbf{1} & 0 & 0 & * & * & * & * & * & * & ? & ? \\
0 & 0 & 0 & \mathbf{1} & 0 & * & * & * & * & * & * & * & ? \\
0 & 0 & 0 & 0 & \mathbf{1} & * & * & * & * & * & * & * & * \\
* & * & * & * & * & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 \\
? & * & * & * & * & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 \\
? & ? & * & * & * & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 \\
? & ? & ? & * & * & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 \\
? & ? & ? & ? & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1}
\end{bmatrix}$$

Band structure matrix algebra:
direct sum of five linear manifolds

$$\begin{pmatrix}
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * & * & * & * & *
\end{pmatrix}$$

Diagonal manifold (actually subalgebra)

$$\begin{pmatrix}
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & *
\end{pmatrix}$$

Upper band manifold

$$
\begin{pmatrix}
0 & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & * & * & * & * & * & * & * & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & * & * & * & * & * & * & * & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & * & * & * & * & * & * & * & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Lower band manifold = (upper band manifold)*

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & * & * & * & * & * & * & * & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & * & * & * & * & * & * & * & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & * & * & * & * & * & * & * & 0
\end{pmatrix}
$$

Upper triangle manifold

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Lower triangle manifold = (upper triangle manifold)*

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Can be considered for **more general algebras** with **involution** and **unit** element
(via abstract general scheme)

Applications:

- Scalar matrix completion (positive / strictly contractive)
- Operator matrix completion (positive / strictly contractive)
- Carathéodory-Toeplitz extension problem
- Nevanlina-Pick interpolation
- Nehari extension problem

Major contributions Kaashoek et al

Briefly discuss here

- **rational contractive interpolants**

Related to '**Nehari**'

Involves State Space method (again)

Rationality requirement: important for concrete applications (system / control theory)

M rational $m \times m$ matrix function

Assumptions:

- M has its poles in the open unit disc $\mathbb{D}$
- M analytic at ∞ and vanishes there

Implies existence **stable** realization

$$M(\lambda) = C(\lambda I_n - A)^{-1}B, \qquad \sigma(\mathbf{A}) \subset \mathbb{D}$$

Strictly contractive rational interpolant F:

- F **rational** without poles on $\mathbb{T}$

- $\|F(\zeta)\| < 1$ for all ζ in the unit circle $\mathbb{T}$

- $F - M$ **analytic** on the open unit disc $\mathbb{D}$

Given **stable** realization

$$M(\lambda) = C(\lambda I_n - A)^{-1}B, \qquad \sigma(\mathbf{A}) \subset \mathbb{D}$$

Introduce:

Controllability Gramian: $G_c = \sum_{j=0}^{\infty} A^j BB^*(A^*)^j$

Observability Gramian: $G_o = \sum_{j=0}^{\infty} (A^*)^j C^*CA^j$

Well-defined because $\sigma(A) \subset \mathbb{D}$ **(stability)**

M has a strictly contractive interpolant

$$\Updownarrow$$

$\sigma(G_c G_o) \subset \mathbb{D}$

Description of all strictly contractive (**rational**) in terms of A, B and C

Identification of a unique one that maximizes an entropy type integral:
the **maximum entropy interpolant** of M

Marinus A. Kaashoek:

central figure in Operator Theory

Earlier slide with MathScinet List co-authors Printscreen

Count: **fifty**

Services/international (selection):

- Several editorships
- Several co-editorships special issues/volumes
- Co-organizer several conferences
- Member/chairman Steering Committees MTNS and **IWOTA**

Will step down by the end of the year

– after becoming eighty in November!

Services/national (selection):

• Chairman Board Dutch Mathematical Society

• Dean Faculty of Mathematics and Computer Science VU Amsterdam

• Dean Faculty of Mathematics Sciences VU Amsterdam

• Netherlands Coordinator European Research Network Analysis and Operators

• Member/chairman important advisory committees Dutch mathematics

Seminar Operator Theory / Analysis VU Amsterdam

Started **1976** . . . **approximately 25 years**

Every Thursday morning

Students, colleagues

Virtually all leading figures Operator Theory

Enormous stimulus

Number of PhD's: 17

Not to forget:

brought Israel Gohberg to Amsterdam on a systematic basis

To quote **Gerard Reve** (Dutch writer, 1923-2006),

the closing sentence of his famous book **The Evenings**:

(Dutch original: *De Avonden*)

"It has been seen, it has not gone unnoticed."

(Dutch: *Het is gezien het is niet onopgemerkt gebleven.*)

Honors!

Honors International:

- Toeplitz Lecturer, Tel-Aviv, 1991

- Member of the Honorary Editorial Board of the journal Integral Equations and Operator Theory, 2008

Doctor Honoris Causa North-West University (Potchefstroom)
South Africa, 2014

Honors National:

- Honorary member Royal Dutch Mathematical Society, 2016
- Royal decoration: Order of the Dutch Lion

Knight in the Royal Order of the Dutch Lion
November 2002

Rien:

Thanks for what you did for mathematics

Thanks for what you did for IWOTA

And, from the **personal** side:

Thanks for having been my teacher, and for becoming my friend!

Thank you

for

your attention!