CHEMNITZ UNIVERSITY OF TECHNOLOGY

Department of Mathematics
Analysis – Inverse Problems

Chemnitz Symposium on Inverse Problems 2010

Conference Guide

September 23 – 24, 2010

Chemnitz, Germany

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Imprint

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General information

Goal

Our symposium will bring together experts from the German and international 'Inverse Problems Community' and young scientists. The focus will be on ill-posedness phenomena, regularization theory and practice, and on the analytical, numerical, and stochastic treatment of applied inverse problems in natural sciences, engineering, and finance.

Location

Chemnitz University of Technology
Straße der Nationen 62 (Böttcher-Bau)
Conference hall 'Altes Heizhaus'
09111 Chemnitz, Germany

Selection of invited speakers

Martin Hanke (Mainz, Germany), opening talk
Thorsten Hohage (Göttingen, Germany)
Stefan Kindermann (Linz, Austria)
Robert Plato (Siegen, Germany)
Elena Resmerita (Linz, Austria)
Masahiro Yamamoto (Tokyo, Japan)

Scientific board

Bernd Hofmann (Chemnitz, Germany)
Peter Mathé (Berlin, Germany)
Sergei V. Pereverzyev (Linz, Austria)
Ulrich Tautenhahn (Zittau, Germany)
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Timetable

Overview for Thursday, September 23

09.00–09.05	**Opening**
09.05–10.45	**Session 1** M. Hanke, E. Resmerita, M. Yamamoto
10.45–11.05	**Coffee break**
11.05–12.20	**Session 2** P. Mathé, U. Tautenhahn, R. Plato
12.20–13.30	**Lunch break**
13.30–15:10	**Session 3** H. Egger, T. Hein, M. A. Iglesias, T. Lahmer
15.10–15.25	**Coffee break**
15.25–16:40	**Session 4** M. Meyer, G. Hu, G. Wachsmuth
16.40–16.50	**Break**
16.50–18:00	**Session 5** B. Hofmann, J. Flemming, N. Rückert, Y. Shao
18.15	**Excursion**

Overview for Friday, September 23

09.00–10.30	**Session 1** T. Hohage, S. Kindermann, B. Kaltenbacher
10.30–10.50	**Coffee break**
10.50–12.05	**Session 2** H. Harbrecht, C. Clason, T. Raasch
12.05–13.15	**Lunch break**
13.15–14:30	**Session 3** K. S. Kazimierski, A. Cornelio, C. Böckmann
14.30–14.40	**Coffee break**
14.40–16:00	**Session 4** N. Puthanmadam Subramaniyam, U. Aßmann, N. Togobytska, R. Engbers
16.00–16.05	**Break**
16.05–17:20	**Session 5** E. Loli Piccolomini, M. Schlottbom, J. Müller, D. Roch

Program for Thursday, September 23

09.00–09.05	**Opening**
09.05–09.45	**Martin Hanke** (Mainz, Germany) *The regularizing Levenberg-Marquardt scheme*
09.45–10.15	**Elena Resmerita** (Linz, Austria) *Morozov principle for an augmented Lagrangian method for solving ill-posed problems*
10.15–10.45	**Masahiro Yamamoto** (Tokyo, Japan) *Inverse problems for Navier-Stokes equations*
10.45–11.05	**Coffee break**
11.05–11.30	**Peter Mathé** (Berlin, Germany) *Regularization under general noise assumptions*
11.30–11.55	**Ulrich Tautenhahn** (Zittau, Germany) *On the interpolation method for deriving conditional stability estimates in ill-posed problems*
11.55–12.20	**Robert Plato** (Siegen, Germany) *The regularizing properties of the trapezoidal method for weakly singular Volterra integral equations of the first kind*
12.20–13.30	**Lunch break**
13.30–13.55	**Herbert Egger** (Graz, Austria) *On model reduction and unique solvability for fluorescence diffuse optical tomography*
13.55–14.20	**Torsten Hein** (Chemnitz, Germany) *Iterative regularization of Landweber-type in Banach spaces*
14.20–14.45	**Marco A. Iglesias** (Cambridge, Massachusetts, USA) *Level-set techniques for facies identification in reservoir modeling*

14.45–15.10	**Tom Lahmer** (Weimar, Germany) *Design of Experiments for Ill-Posed Problems With Application to Water Dam Monitoring*
15.10–15.25	**Coffee break**
15.25–15.50	**Marcus Meyer** (Chemnitz, Germany) *Parameter identification in nonlinear elasticity – theory, results, and problems*
15.50–16.15	**Guanghui Hu** (Berlin, Germany) *Uniqueness in Inverse Scattering of Elastic Waves by Doubly Periodic Structures*
16.15–16.40	**Gerd Wachsmuth** (Chemnitz, Germany) *Regularization results for inverse problems with sparsity functional*
16.40–16.50	**Break**
16.50–17.15	**Bernd Hofmann** (Chemnitz, Germany) *Some new aspects of regularization in the context of variable Hilbert scales*
17.15–17.35	**Jens Flemming** (Chemnitz, Germany) *Variational inequalities versus source conditions in Hilbert spaces*
17.35–17.50	**Nadja Rückert** (Chemnitz, Germany) *Some studies on regularization of Poisson distributed data*
17.50–18.00	**Yuanyuan Shao** (Zittau/Chemnitz, Germany) *Generalized discrepancy principle for ill-posed problems with noisy data*
18.15	**Excursion to ‘Rabensteiner Felsendome’ with conference dinner** departure 18.15 by bus at hotel ‘Chemnitzer Hof’

Program for Friday, September 24

09.00–09.30	**Thorsten Hohage** (Göttingen, Germany) *Inverse problems in photonic imaging*
09.30–10.00	**Stefan Kindermann** (Linz, Austria) *On the convergence of heuristic parameter choice rules*
10.00–10.30	**Barbara Kaltenbacher** (Graz, Austria) *Regularization by Local Averaging Regression*
10.30–10.50	**Coffee break**
10.50–11.15	**Helmut Harbrecht** (Stuttgart, Germany) *An efficient numerical method for a shape identification problem arising from the heat equation*
11.15–11.40	**Christian Clason** (Graz, Austria) L^1 *data fitting for nonlinear inverse problems*
11.40–12.05	**Thorsten Raasch** (Mainz, Germany) *Optimal convergence rates of* ℓ_1*-constrained Tikhonov regularization under compressibility assumptions*
12.05–13.15	**Lunch break**
13.15–13.40	**Kamil S. Kazimierski** (Bremen, Germany) *On Engl's discrepancy principle*
13.40–14.05	**Anastasia Cornelio** (Modena, Italy) *Regularized Nonlinear Least Squares Methods for Hit Position Reconstruction in Small Gamma Cameras*
14.05–14.30	**Christine Böckmann** (Potsdam, Germany) *Levenberg-Marquardt Method under Logarithmic Source Condition*
14.30–14.40	**Coffee break**

14.40–15.00	**Narayan Puthanmadam Subramaniyam** (Tampere, Finland) *Regularization methods for inverse EEG problems*
15.00–15.20	**Ute Aßmann** (Duisburg, Germany) *Identification of an unknown parameter in the main part of an elliptic PDE*
15.20–15.40	**Nataliya Togobytska** (Berlin, Germany) *An inverse problem for laser-induced thermotherapy arising in tumor tissue imaging*
15.40–16.00	**Ralf Engbers** (Münster, Germany) *Nonlinear Inverse Problem of Myocardial Blood Flow Quantification*
16.00–16.05	**Break**
16.05–16.25	**Elena Loli Piccolomini** (Bologna, Italy) *A feasible direction method for the solution of an inverse ill-posed problem*
16.25–16.45	**Matthias Schlottbom** (Aachen, Germany) *Analysis and regularization in diffuse optical tomography*
16.45–17.05	**Jahn Müller** (Münster, Germany) *Total Variation Regularization in 3D PET Reconstruction*
17:05–17.20	**Diana Roch** (Chemnitz, Germany) *Studies on convolution equations*

Abstracts

Identification of an unknown parameter in the main part of an elliptic PDE

Ute Aßmann, Arnd Rösch

We are interested in identifying an unknown material parameter $a(x)$ in the main part of an elliptic partial differential equation

$$-\text{div }(a(x) \text{ grad } y(x)) = g(x) \text{ in } \Omega$$

with corresponding boundary conditions. We discuss a Tichonov regularization

$$\min_a J(y,a) = \|y - y_d\|^2_{L^2(\Omega)} + \alpha \|a\|^2_{H^s(\Omega)}$$

with $s > 0$. Moreover, we require the following constraints for the unknown parameter

$$0 < a_{min} \leq a(x) \leq a_{max}.$$

The talk starts with results on existence of solutions and necessary optimality conditions. The main part of the talk will be devoted to sufficient optimality conditions.

Levenberg-Marquardt Method under Logarithmic Source Condition

Christine Böckmann

We regard a general - possibly nonlinear and ill-posed - operator equation $F(x) = y$, where the operator $F : \mathcal{D}(F) \to Y$ is Fréchet differentiable on its domain $\mathcal{D}(F) \subset X$ and X, Y are Hilbert spaces. We assume that the exact data y is attainable and there exists an exact solution $x^\dagger \in \mathcal{D}(F)$ (which need not to be unique) with $F(x^\dagger) = y$. However, we have only noisy data y^δ by hand satisfying $\|y^\delta - y\| \le \delta$. A lot of applications, e.g. heat conduction and potential theory, are severely ill-posed. For such problems logarithmic source conditions have natural interpretations whereas classical Hölder source conditions are far too restrictive. Using logarithmic source condition instead of the Hölder type one,

$$f_p(\lambda) := \begin{cases} (\ln \frac{\mathrm{e}}{\lambda})^{-p} & \text{for } 0 < \lambda \le 1,\, p > 0 \\ 0 & \text{for } \lambda = 0 \end{cases},$$

we show logarithmic convergence rate of the Levenberg-Marquardt method

$$x_{n+1}^\delta = x_n^\delta + (F'[x_n^\delta]^* F'[x_m^\delta] + \alpha_n I)^{-1} F'[x_n^\delta]^* (y^\delta - F(x_n^\delta))$$

under appropriated properties of F as well as Morozov's discrepancy principle. Finally, we apply the method to recover the shape of a homogeneous mass distribution from the knowledge of measurements of its gravitational potential.

L^1 data fitting for nonlinear inverse problems

Christian Clason, Bangti Jin

This talk is concerned with L^1 data fitting for nonlinear inverse problems, which is advantageous if the data is corrupted by impulsive noise. In particular, we are interested in the minimization problem

$$\min_{u\in L^2} \|S(u) - y^\delta\|_{L^1} + \frac{\alpha}{2}\|u\|_{L^2}^2,$$

where $u \in L^2$ is the unknown parameter, $y^\delta \in L^\infty$ represents experimental measurements corrupted by (impulsive) noise, and S is the parameter-to-observation mapping. Even if S has sufficient differentiability and continuity properties, the problem is not differentiable and lacks local uniqueness, which makes its numerical solution challenging.

In this talk, we discuss approximation properties of the minimizers to nonlinear functionals with L^1 data fitting and suggest a strategy for selecting the regularization parameter based on a balancing principle. We also introduce a regularized primal-dual formulation of this problem, for which local uniqueness can be shown under a (reasonable) second order sufficient condition. The same condition permits the application of a superlinearly convergent semi-smooth Newton method for the numerical solution of the discretized problem.

We illustrate our approach through the model problem of recovering the potential in an elliptic boundary value problem from distributed observational data.

Regularized Nonlinear Least Squares Methods for Hit Position Reconstruction in Small Gamma Cameras

Anastasia Cornelio

To improve the spatial resolution of a gamma camera and consequently the quality of the image reconstruction, it's fundamental to accurately reconstruct the photon hit position on the detector surface. The methods proposed in literature to estimate the hit position work well at the center of the detector but tipically deteriorate near the edges. The increasing demand of *small* PET systems with very high performance and the consequent necessity to recover the information near the edges are the motivations of this work.

We apply iterative optimization methods based on regularization of the nonlinear least squares problem to estimate the hit position. The idea is to find a model f, depending on unknown parameters $z \in \mathbb{R}^n$ (with n tipically less than 5), that well describes the light distribution, produced by the photon impact on the detector, and then to use the least squares method to choose the parameters z that best fit f to the observed data v. We state the problem as a nonlinear least squares problem

$$\min_z \mathcal{F}(z) = \frac{1}{2}\|f(z) - v\|_2^2. \qquad (*)$$

The problem is ill conditioned so it needs regularization to obtain meaningful solutions in presence of noise. We apply regularization either by adding a regularization term to the objective function $\mathcal{F}$, or by solving $(*)$ with an iterative regularization method, in which the number of iterations is made to play the role of regularization parameter, in order to avoid the semiconvergent behaviour of the solution. Newton-type and descend methods are applied to solve this regularized nonlinear problem.

Numerical results show that the proposed methods allow to obtain good results with a very small number of iterations, and consequently with a low computational cost, and to recover information near the edges of the detector.

On model reduction and unique solvability for fluorescence diffuse optical tomography

Herbert Egger

Fluorescence tomography is a medical imaging technique based on the following measurement principle: (i) an object is illuminated by a light source; (ii) a part of the excitation light is absorbed by fluorophores and re-emitted at a longer wavelength; (iii) the emitted light is detected at the boundary. This process is governed by a coupled system of elliptic partial differential equations. The goal of fluorescence tomography is to obtain cross-sectional images of the fluorophore distribution inside the body by "inversion" of the measurements.

In this talk, we discuss certain simplifications of the forward problem and derive error estimates for the error introduced by these approximations. We illustrate, that the model reductions have advantages, both, from a numerical point of view, but also for the analysis of the inverse problem. In particular, we are able to establish unique solvability for one of the reduced models.

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Nonlinear Inverse Problem of Myocardial Blood Flow Quantification

Ralf Engbers

Dynamic positron emission tomography (PET) allows for noninvasive examination of physiological processes. Radioactive water ($H_2{}^{15}O$) as a PET-tracer is the preferred candidate for examining myocardial blood flow because of its short half-time, resulting in a low radiation burden to the patient, and its high diffusibility. Unfortunately, the short half-time leads to noisy, low-resolution reconstructions.

The common approach for this quantification problem is to reconstruct images for each temporal dataset independently via the standard EM-algorithm or FBP and to compute the parameters from these images. However, the temporal correlation between the datasets is neglected in this approach.

Rather than using the correlation between noisy, low resolution images we want to use the temporal correlation inherent in the datasets. This can be achieved by building up a nonlinear physiological model depending on physiological parameters (e.g. perfusion) and solving the respective parameter identification problem. As another advantage, regularization can be added to each parameter independently to ensure meaningful results. A forward-backward operator splitting method can be used to solve this inverse problem numerically.

References

[1] M. Benning. A Nonlinear Variational Method for Improved Quantification of Myocardial Blood Flow Using Dynamic $H_2{}^{15}O$ PET, 2008, Diploma Thesis.

Variational inequalities versus source conditions in Hilbert spaces

Jens Flemming

For proving convergence rates of regularized solutions of ill-posed operator equations one has to pose assumptions on the 'smoothness' of the exact solution. We discuss four concepts for expressing solution smoothness:

- source conditions,
- approximate source conditions,
- variational inequalities,
- approximate variational inequalities.

The focus will lie on the third one. Variational inequalities have been introduced in a Banach space setting in 2007 (see [1]) and several extensions were developed by the Chemnitz group (see, e.g., [2,3]). To understand this powerful concept (cf. [4]) we concentrate on Hilbert space situations and show that the amount of information contained in a variational inequality is exactly the same as for approximate source conditions. The proof of this fact is based on the (quite technical) idea of approximate variational inequalities. It turns out that variational inequalities can be interpreted as Fenchel dual formulations of source conditions.

Applying the techniques to linear operators in Banach spaces motivates extended use of Bregman distances instead of norms.

References

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[5] J. Flemming: *Solution smoothness of ill-posed equations in Hilbert spaces: four concepts and their cross connections.* 2010, submitted.

The regularizing Levenberg-Marquardt scheme

Martin Hanke

In 1997 we introduced a variant of the Levenberg-Marquardt method for nonlinear ill-posed problems. Like the original Levenberg-Marquardt method this method modifies the Gauss-Newton iteration by searching in each iteration the local update within a certain trust region. In contrast to the usual definition of a trust region we proposed to choose the update with minimal norm that reduces the linearized residual by a certain (prescribed) amount. In this talk we show that if the iteration is terminated according to the discrepancy principle then the method is a regularization method with optimal order accuracy – under standard assumptions on the nonlinearity of the underlying operator equation.

An efficient numerical method for a shape identification problem arising from the heat equation

Helmut Harbrecht, Johannes Tausch

The present talk is dedicated to the determination of the shape of a compactly supported constant source in the heat equation from measurements of the heat flux through the boundary. This shape identification problem is formulated as the minimization of a least-squares cost functional for the desired heat flux at the boundary. The shape gradient of the shape functional under consideration is computed by means of the adjoint method. A gradient based nonlinear Ritz-Galerkin scheme is applied to discretize the shape optimization problem. The state equation and its adjoint are computed by a fast space-time boundary element method for the heat equation. Numerical experiments are carried out show the feasibility of the present approach.

Iterative regularization of Landweber-type in Banach spaces

Torsten Hein

Let X and Y denote two Banach spaces. We consider the linear ill-posed operator equation

$$A\,x = y \qquad x \in X,\ y \in Y,$$

where $A : X \longrightarrow Y$ describes a linear bounded operator which non-closed range $\mathcal{R}(A)$. For $\delta > 0$ and given noisy data $y^\delta \in Y$ with knowing bound $\|y^\delta - y\| \le \delta$ for the noise level we deal with the Landweber iteration approach

$$\begin{aligned} x_0^\delta &:= x_0 \in X,\ x_0^* := J_s(x_0^\delta) \\ x_{n+1}^* &:= x_n^* - \mu_n \psi_n^* \\ x_{n+1}^\delta &:= J_s^{-1}(x_{n+1}^*) \end{aligned}$$

together with the discrepancy principle as stopping criterion for the iteration process. Here, $\psi_n^* \in X^*$ describes either the gradient of the functional $x \mapsto \frac{1}{p}\|A\,x - y^\delta\|^p$ or a modified variant. Moreover, $p, s \in (1, \infty)$ and J_s denotes the duality mapping from the space X into its dual space X^* with gauge function $t \mapsto t^{s-1}$.

In order to achieve a tolerable speed of convergence of the algorithm we have to apply a proper choice of the step size parameter μ_n in each iteration. Motivated by the method of minimal error (in Hilbert spaces) and taking into account the noisy data we derive a one-dimensional minimization problem for calculating the (optimal) parameter μ_n in each iteration. We present convergence and stability results of the methods under consideration.

Finally we give a short numerical example which illustrates the efficiency of the presented algorithms.

Some new aspects of regularization in the context of variable Hilbert scales

Bernd Hofmann

In this talk, we discuss the cross connections between different approaches to convergence rates in regularization for linear ill-posed operator equations in Hilbert spaces using variable Hilbert scales. It is well-known that the concept developed by Tautenhahn (see [2]) and the concept by Mathé and Pereverzev (see [3]) lead to comparable results in essential points even if the derived formulae and the required convexity/concavity conditions have different structure. In classical regularization theory for linear ill-posed problems presented in the monographs by Vainikko et al. 1986 and Engl et al. 1996 the qualification of a regularization method is a positive real number or infinity characterizing the upper limit of order optimality occurring for the method. In the context of recent progress in regularization theory arising from the stringent consideration of general source conditions and variable Hilbert scales with index functions a more sophisticated qualification concept was introduced (see [3] and the more recent papers [5] and [6]). Qualification is now a function-valued concept for any linear regularization method and characterizes appropriate approximation properties of the method with respect to the regularization parameter. As was shown recently in [7], for every element in a Hilbert space and every positive self-adjoint and injective linear operator there is an index function for which a general source condition holds true. This allows us to establish the variable Hilbert scale approach in regularization as an all-embracing tool for obtaining convergence rates. By using link conditions like range inclusions (see [4] and [5]) such approach can be extended to a wider field of a priori information concerning the expected solution. It was an open problem whether the conditions of the early published concept by Hegland (see [1]) plays another role. Along the lines of the recent paper [8] we can show now that the basic results and conditions of all three mentioned concepts coincide, but their shapes and their potential for interpretation are quite different. Parts of the talk refer to joint work with Peter Mathé (WIAS Berlin) and Markus Hegland (ANU Canberra).

References

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Inverse problems in photonic imaging

Thorsten Hohage

Photonic imaging is a growing research field with huge impact on the life sciences including techniques such as fluorescence microscopy, x-ray imaging using phase retrieval, and positron emission tomography. A common theme is that data are given in the form of photon counts of an array of detectors, and an inverse problem needs to be solved to reconstruct the desired quantity. The number of counts in each detector is a Poisson distributed random variable. We introduce Poisson processes as natural continuous framework for describing photon count data. Moreover, we present regularization methods for linear and nonlinear inverse problems involving the Kullback-Leibler divergence as natural data misfit functional and show convergence and convergence rates as the total number of counts tends to infinity. We conclude with some real data examples from 4Pi fluorescence microscopy and x-ray optics.

Uniqueness in Inverse Scattering of Elastic Waves by Doubly Periodic Structures

Guanghui Hu, Johannes Elschner

This talk is concerned with the inverse scattering of a time-harmonic elastic wave by an unbounded doubly-periodic structure in $\mathbb{R}^3$. Such structures are also called diffraction gratings and have many important applications in diffractive optics, radar imaging and non-destructive testing.

We assume that a polyhedral diffraction grating divides the three-dimensional space into two non-locally perturbed half-spaces filled with homogeneous and isotropic elastic media, and that a time-harmonic pressure or shear wave is incident on the grating from above. Furthermore, the grating is supposed to have an impenetrable surface on which normal stress and tangential displacement (resp. normal displacement and tangential stress) vanish. This gives rise to the so-called the third (resp. fourth) kind boundary conditions for the Navier equation. We firstly show some solvability results on the direct scattering problem in Lipschitz domains; and then classify all the grating profiles that can not be uniquely determined from a knowledge of the scattered field measured above the grating; finally, we demonstrate some examples of the unidentifiable gratings for illustrating that, in general case, one incident elastic wave fails to uniquely determine a doubly periodic structure under the boundary conditions of the third or fourth kind.

The main tool we used is the reflection principle for the Navier equation, which was recently established by Elschner J. and Yamamoto M. [Inverse Problems, 26 (2010) pp. 045005/1–045005/8]. Relying on such principle, we prove that the total fields are analytic functions in $\mathbb{R}^3$ and remain rotational and reflectional invariance, if they are generated by two different gratings and have the same measurement. This enables us to determine and classify all the unidentifiable grating profiles corresponding to each incident elastic wave, even in the resonance case where a Rayleigh frequency is allowed.

This is a joint work with Johannes Elschner at WIAS.

Level-set techniques for facies identification in reservoir modeling

Marco A. Iglesias, Dennis McLaughlin

In this talk we report the application of level-set techniques for reservoir facies identification [4]. This ill-posed inverse problem is formulated as a shape optimization problem, where the aim is to find a region (a geologic facies) that minimizes the misfit between predictions and measurements from a subsurface petroleum reservoir. The shape optimization problem is constrained by a large-scale nonlinear system of PDE's that model multiphase (oil-water) flow in the reservoir. This model is converted to a weak (integral) form to facilitate the application of standard results for the computation of shape derivatives. The shape derivatives are needed to apply the iterative level-set solution approach developed by Burger in [1,2]. This approach describes the unknown facies shape with a level-set function that is modified through a sequence of geometrical deformations. The shape derivatives of the reservoir model define a level-set velocity that insures that the new shape constructed in each step of the iterative sequence decreases the data misfit. We present results for the identification of geologic facies derived with both the gradient-based (GB) approach of [1] and the Levenberg-Marquardt (LM) approach of [2]. Our adjoint formulation makes application of the GB approach straightforward. The LM technique requires the solution of a large-scale Karush-Kuhn-Tucker system of equations at each iteration of the scheme. We solve this KKT system with a representer-based approach proposed by [3]. We present experiments to show and compare the capabilities and limitations of the proposed implementations. When the well configuration is adequate, both level-set techniques are able to give facies estimates that recover the main features of the true facies distribution. In relevant cases where the initial shape undergoes substantial changes to recover the true shape, our representer-based implementation of the LM technique outperforms the efficiency of the GB approach.

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Regularization by Local Averaging Regression

Barbara Kaltenbacher, Harro Walk

In this talk we consider combination of the ideas of regression function estimation on one hand and regularization by discretization on the other hand to a regularization method for linear ill-posed problems with additive stochastic noise. A general convergence result is provided and its assumptions are verified for the partitioning estimators and to some extent for kernel estimators. As an example of an inverse problem we consider Volterra integral equations of the first kind.

On Engl's discrepancy principle

Kamil S. Kazimierski

In this talk we present a new choice rule for the parameter α of the Tikhonov functional

$$\mathrm{T}_\alpha(x) := \tfrac{1}{p}\|Ax - y^\delta\|_Y^p + \alpha\tfrac{1}{q}\|x\|_X^q \qquad p, q > 1,$$

where A is a linear, continuous operator mapping between the Banach spaces X and Y.

For the *a-priori* choice rule $\alpha(\delta) \sim \delta^\kappa$ a convergence rate of the form

$$D_{j_q^X}(x^\dagger, x^\delta_{\alpha(\delta)}) \sim \delta^\nu$$

for the Bregman distance $D_{j_q^X}$ with

$$0 < \nu \leq \tfrac{4}{3}$$

can be proven (under appropriate source conditions).

However, one strives for an *a-posteriori* parameter choice rule which attains similar rates. Often the discrepancy principle of Morozov is used as such rule, i.e. one chooses $\alpha(\delta, y^\delta)$ such that $\|Ax^\delta_{\alpha(\delta,y^\delta)} - y^\delta\| \sim \delta$. However, to the authors best knowledge then at best rates with $0 < \nu \leq 1$ can be proven.

In this talk we will discuss another discrepancy based parameter choice rule, where $\alpha(\delta, y^\delta)$ is chosen such that

$$\|A^* j_p^Y(Ax^\delta_{\alpha(\delta,y^\delta)} - y^\delta)\|_{X^*}^{q^*} \sim \delta^r \alpha^{-s},$$

with the duality mapping j_p^Y and appropriately chosen parameters r, s. We remark that this rule was proposed originally by Engl for linear operators mapping between two Hilbert spaces. We will show that Engl's approach can also be extended to linear operators mapping between Banach spaces, where the same rates as for the *a-priori* parameter choice rules can be obtained.

On the convergence of heuristic parameter choice rules

Stefan Kindermann

We study convergence properties of minimization based heuristic (or noise-level free) parameter choice rules in the regularization theory of ill-posed problems. According to a result by Bakushinskii any worst-case convergent parameter choice rule has to take into account the noise level. For a convergence analysis, it is therefore essential to put additional conditions on the noise or on the solution to obtain convergence results. It turns out that under reasonable conditions convergence, convergence rates and oracle type estimates for many methods can be proven. Within this analysis we compare several different rules in view of theoretical convergence and convergence rates with different regularizations.

Design of Experiments for Ill-Posed Problems With Application to Water Dam Monitoring

Tom Lahmer

The aim of this research is to perform an optimal experimental design for the reliable estimation of damages for water dams. The safe operation of dams, dikes or embankments requires continuous monitoring in order to detect any changes concerning the statical structure. Damages which may result from cyclic loadings, variations in temperature, ageing, chemical reactions and so on need to be identified as fast and as reliable as possible. The basis for these investigations is a hydro-mechanically coupled model with heterogeneous material distributions in which damages are described by a smeared crack model. In the case of damages, the changes of the main parameters in a multifield model are strongly correlated. This correlation is particularly considered during the inverse analysis, which is the detection of the damages from combined hydro-mechanical data.

Iterative regularising methods are applied to solve the nonlinear inverse problem.

For an efficient monitoring, an optimal experimental design framework for nonlinear ill-posed problems is derived. The design problem is formulated in that manner, that it proposes locations for the sensors that guarantee a reliable identification of the damages (small variances) keeping the bias introduced by the regularisation as low as possible.

The methodology, solutions of the coupled forward and inverse problem as well as numerical results of the design process will be presented during the talk.

A feasible direction method for the solution of an inverse ill-posed problem

Elena Loli Piccolomini, G. Landi

Linear inverse ill-posed problems arising, for example, from the discretization of Fredholm integral equations, are of the form:

$$Ax = z$$

where $A \in \mathbb{R}^{n\times n}$ is the discretization of the integral kernel, $z \in \mathbb{R}^n$ is the data vector usually affected by noise with variance σ^2 and $x \in \mathbb{R}^n$ is the solution vector.

For the problem solution we consider the following constrained optimization formulation:

$$\text{minimize } f(x) \qquad \text{subject to} \qquad x \in \Omega$$

where $f(x)$ is a continuously differentiable regularization function from $\mathbb{R}^n$ into $\mathbb{R}$ and the feasible set Ω is the sphere of radius σ and center $z \in \mathbb{R}^n$ defined by

$$\Omega = \{x \in \mathbb{R}^n | \|x - z\|^2 \leq \sigma^2\}.$$

We propose, for the solution of the problem, a method combining ideas from feasible direction and trust region methods (we call it FDTR method). It generates strictly feasible iterates having the general form $x_{k+1} = x_k + \lambda_k d_k$, where d_k is the search direction and λ_k is the step-length. The direction d_k is determined by inexactly solving a trust region subproblem consisting in minimizing the classical second-order model of $f(x)$ around the current iterate x_k, subject to a quadratic constraint ensuring the feasibility of the next iterate. The step-length is computed by the Armijo rule in order to guarantee a sufficient decrease of the objective function. We prove that the method is globally convergent under standard assumptions.

Some numerical tests have been performed in image deblurring (where A is the Point Spread Function) and denoising (where $A = I$). As a regularization function f we have considered both the Tikhonov and the Total Variation functions. The numerical tests show the effectiveness of the method both for its accuracy and its computational cost.

Regularization under general noise assumptions

Peter Mathé, Ulrich Tautenhahn

We explain how the major results which were obtained recently in P P B Eggermont, V N LaRiccia, and M Z Nashed. *On weakly bounded noise in ill-posed problems*, **Inverse Problems**, 25(11):115018 (14pp), 2009, can be derived from a more general perspective of recent regularization theory. By pursuing this further we provide a general view on regularization under general noise assumptions, including weakly and strongly controlled noise.

This is joint work with Ulrich Tautenhahn, Zittau.

Parameter identification in nonlinear elasticity – theory, results, and problems

Marcus Meyer

We consider parameter identification problems arising in nonlinear elasticity, whereas we assume that from measured deformation data of a loaded elastic body the corresponding material properties in terms of some material parameters need to be identified. While solving those inverse problems in a practical framework, several crucial problems emerge, as e.g. questions concerning existence, uniqueness, and regularity of solutions, or the implementation of efficient numerical methods, which is due to the involved nonlinearities an essential issue.

In the first part of the talk we present a survey of elasticity theory and the corresponding nonlinear PDE model and introduce solution approaches for the inverse problem basing on nonlinear constrained optimization methods. The second part of the presentation is devoted to a discussion of the interplay of several problems and open questions concerning the analysis and the numerics of the inverse problem.

Total Variation Regularization in 3D PET Reconstruction

Jahn Müller

Due to the application of tracers with a short radioactive half-life (e.g. radioactive water $H_2{}^{15}O$) in positron emission tomography (PET), one obtains images with bad Poisson statistics (low count rates).

To achieve reasonable results from this data, one may be interested in reconstructing at least major structures with sharp edges, which can also be a prerequisite for further processing, e.g. segmentation of objects.

The focus is set on reconstruction strategies combining expectation maximization (EM) (cf. [1]) and total variation (TV) (cf. [2]) based regularization. In particular a postprocessing TV denoising algorithm as well as a nested EM-TV algorithm is presented (cf. [3]). In order to guarantee sharp edges, the smoothing of approximate total variation is avoided by using dual (cf. [4]) or primal dual approaches for the numerical solution.

The performance of these approaches is illustrated for data in positron emission tomography, namely reconstructions of cardiac structures with ^{18}F-FDG and $H_2{}^{15}O$ tracers, respectively.

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The regularizing properties of the trapezoidal method for weakly singular Volterra integral equations of the first kind

Robert Plato

The repeated trapezoidal method was considered by P. Eggermont for the numerical solution of weakly singular Volterra integral equations of the first kind with exactly given right-hand sides. In the present talk we review this method and consider its regularizing properties for perturbed right-hand sides. Some numerical results are presented.

Regularization methods for inverse EEG problems

Narayan Puthanmadam Subramaniyam,
Outi Väisänen, Jaakko Malmivuo

Electroencephalography (EEG) is a non-invasive way of measuring the electrical activity of the brain from the scalp using electrodes. The EEG offers excellent temporal resolution, but suffers due to limited spatial resolution. This is due to the volume conduction effects of the tissues present in the human head between the sources and sensors. Estimation of cortical potential distribution from the scalp EEG is an inverse problem which requires regularization techniques which aim to fit data with additional penalization. We compared penalizations based on both L1 and L2 norm. The results show that, in order to get a sparse solution, which is desired in inverse EEG problems, L1 norm is the best approach. The cortical potential distribution with L1-norm had more focal potential distribution around the occipitotemporal and temporal parts of the cortex for EEG data pertaining to face recognition/perception. The cortical potential distribution obtained with L2 norm for the same data was relatively blurred and smooth. With L1 norm, we have obtained accurate and higher resolution cortical potential maps in comparison to L2-norm.We also compared two methods to choose the regularization parameter - the generalized cross validation (GCV) method and L-curve method. Results show that GCV is a more robust method to choose the regularization parameter compared to the L-curve method in cases where the discrete Picard condition is not satisfied.

Optimal convergence rates of ℓ_1-constrained Tikhonov regularization under compressibility assumptions

Thorsten Raasch

We are concerned with the convergence properties of ℓ_1-constrained Tikhonov regularization of linear ill-posed problems $Ax = y$ with noisy data y^δ. Under the assumption that the ideal solution $x^\dagger$ is *sparse*, i.e, it has a finite expansion in some underlying ansatz system, convergence and convergence rates of the regularized solutions to $x^\dagger$ have been addressed in a series of recent papers. However, sparsity assumptions constitute a strong link between the discretization method and the unknown solution, and may therefore be hard to realize in practical applications. In this talk, we adress the question to which extent also *compressible* solutions $x^\dagger$ with fast-decaying expansion coefficients give rise to convergent regularization schemes. Moreover, an analysis from the viewpoint of nonlinear approximation theory seems missing. We investigate under which conditions on the forward operator and on the data the regularized solutions can be expected to constitute quasi-optimal approximations of the ideal solution, when compared to the best N-term approximation. The results are illustrated by several numerical experiments.

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Morozov principle for an augmented Lagrangian method for solving ill-posed problems

Elena Resmerita

The Bregman iteration method for image restoration as proposed by Osher et al in 2005 has been proven to have several advantages over classical imaging methods. Moreover, it has provided inspiration for numerous research ideas related to different challenges, including sparsity problems. It has been recently pointed out that the method is equivalent to an augmented Lagrangian method. This talk presents error estimates for the Morozov principle applied to the iterative procedure for solving linear ill-posed problems, and emphasizes the convergence type and convergence rates for bounded variation and ℓ_1 sparsity settings (joint work with Klaus Frick and Dirk Lorenz).

Studies on autoconvolution equations

Diana Roch

Criteria for choosing regularization parameter are well investigated in the literature for linear ill-posed problems. The problem of autoconvolution is a nonlinear ill-posed problem, but rather closed to the linear case. In our numerical studies we try to investigate the chances and limitations of empirical criteria like L-Curve-Method for such problems.

Some studies on regularization of Poisson distributed data

Nadja Rückert

In some applications, e.g. astronomy, medical applications, images are only available as numbers of photons detected at each pixel. These photon counts are modeled as a Poisson process and therefore imply the generalized Kullback-Leibler divergence as the fitting functional. The aim is to recover the original image from the Poisson distributed data. Some first numerical studies will be presented.

Analysis and regularization in diffuse optical tomography

Matthias Schlottbom, Herbert Egger

In this talk we will investigate the problem of diffuse optical tomography. This is the reconstruction of optical parameters related to absorption and scattering from optical measurements at the boundary. As a transport model for the propagation of light we utilize a second order elliptic equation, i.e. the diffusion approximation which can be derived by a first order approximation to the linear Boltzmann equation. Based on this forward model we will define an appropriate forward operator F which maps optical parameters to boundary measurements. The resulting nonlinear inverse problem is severly-illposed and hence regularization is needed. The main objective of the talk is to analyze properties of F in order to make standard results from regularization theory applicable. Based on $W^{1,p}$ regularity results for solutions of elliptic equations, properties like weak sequential closedness or certain differentiability properties are shown. The talk concludes with application of these results to Tikhonov regularization and a numerical example.

Generalized discrepancy principle for ill-posed problems with noisy data

Yuanyuan Shao

The goal of this talk is to present recent results for solving linear ill-posed problems $A_0 x = y_0$ where $A_0 : X \to Y$ is a bounded operator between Hilbert spaces X and Y. We are interested in problems where

(i) instead of $y_0 \in \mathcal{R}(A_0)$ we have noisy data $y_\delta \in Y$ with $\|y_0 - y_\delta\| \leq \delta$,

(ii) instead of A_0 we have a noisy operator $A_h \in \mathcal{L}(X, Y)$ with $\|A_0 - A_h\| \leq h$.

Since $\mathcal{R}(A_0)$ is assumed to be non-closed, the solution $x^\dagger$ of the operator equation $A_0 x = y_0$ does not depend continuously on the data. Hence, for solving $A_0 x = y_0$ with noisy data (y_δ, A_h) some regularization methods are required. In the present talk we study the method of Tikhonov regularization with differential operators where the regularization parameter is chosen *a posteriori* by the generalized discrepancy principle (GDP, see [1]). Under certain smoothness assumptions for $x^\dagger$ we provide order optimal error bounds that characterize the accuracy of the regularized solution. In addition we discuss computational aspects and provide fast algorithms for the computation of the regularization parameter. These algorithms are globally and monotonically convergent. The results extend earlier results where the operator is exactly given. Some of our theoretical results are illustrated by numerical experiments. This talk is a joint work with Shuai Lu, Sergei Pereverzyev and Ulrich Tautenhahn.

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On the interpolation method for deriving conditional stability estimates in ill-posed problems

Ulrich Tautenhahn

In this talk we consider ill-posed problems $Ax = y$ where A is a linear operator between Hilbert spaces X and Y with non-closed range $\mathcal{R}(A)$ and ask for conditional stability estimates on a given set $M \subset X$. An operator A is called to satisfy a conditional stability estimate on M if there exists a continuous, monotonically increasing function $\beta : \mathbb{R}^+ \to \mathbb{R}^+$ with $\lim_{t\to 0} \beta(t) = 0$ (index function) that obeys

$$\|x_1 - x_2\| \le \beta\left(\|Ax_1 - Ax_2\|\right) \quad \text{for all} \quad x_1, x_2 \in M. \qquad (*)$$

Conditional stability estimates $(*)$ are, e. g., important for the study of following questions: (i) Which best possible error bounds can be obtained for identifying the solution $x^\dagger$ of the operator equation $Ax = y$ from noisy data $y^\delta \in Y$ under the assumptions $\|y - y^\delta\| \le \delta$ and $x^\dagger \in M$? (ii) How to regularize such that the best possible error bounds can be guaranteed? For deriving conditional stability estimates on general source sets $M_{\varphi,E} = \left\{x \in X \mid x = [\varphi(A^*A)]^{1/2}v,\ \|v\| \le E\right\}$ with some index function φ we use interpolation techniques in variable Hilbert scales that allow to derive the formula

$$\beta(\delta) = E\sqrt{\varrho^{-1}(\delta^2/E^2)} \quad \text{with} \quad \varrho(t) = t\varphi^{-1}(t) \qquad (**)$$

in case ϱ is convex. Due to this formula, our *interpolation method for deriving conditional stability estimates on sets M* consists in executing following three steps:

(i) Derive the index function φ such that the set M coincides with the set $M_{\varphi,E}$.

(ii) Compute the function $\varrho(t) = t\varphi^{-1}(t)$ and prove its convexity.

(iii) Derive a formula for β given by $(**)$.

We apply this method to different inverse PDE problems and show that, depending on different subsets $M \subset X$, the conditional stability estimates $(*)$ may be of Hölder type, of logarithmic type or of some other type.

An inverse problem for laser-induced thermotherapy arising in tumor tissue imaging

Nataliya Togobytska

Laser-induced thermotherapy (LITT) is an advanced technique for cancer treatments, which is of minimally invasion and especially applicable for patients with liver metastases. In this method laser light is diffused in tumorous tissue leading to an increase in temperature and subsequent coagulation (i.e destruction) of the tumour tissue.The treatment is guided using magnetic resonance imaging (MRI). Unfortunately, MRI is known to have either a good spatial or a good temporal resolution, making it difficult to predict the final size of the coagulated zone. Hence, there is a strong demand for computer simulations of LITT to support therapy planning and finding an optimal dosage. The mathematical model for the LITT consists of a bio-heat equation for the temperature distribution in the tissue and an ordinary differential equation to describe the evolution of the coagulated zone. In this talk i will briefly introduce LITT and the mathematical model and finally discuss the problem of identification of the temperature dependent growth parameter of the coagulated tissue from the temperature measurement data.

Regularization results for inverse problems with sparsity functional

Gerd Wachsmuth, Daniel Wachsmuth

We consider an optimization problem of the type

$$\begin{cases} \text{Minimize} & F(u) = \frac{1}{2}\|\mathcal{S}u - z_\delta\|_H^2 + \alpha\,\|u\|_{L^2(\Omega)}^2 + \beta\,\|u\|_{L^1(\Omega)} \\ \text{such that} & u \in U_{\text{ad}} \subset L^2(\Omega). \end{cases} \tag{$\mathbf{P}_{\alpha,\delta}$}$$

Here, $\Omega \subset \mathbb{R}^n$ is a bounded domain, H is some Hilbert space, $\mathcal{S} \in \mathcal{L}(L^2(\Omega), H)$ compact (e.g. the solution operator of an elliptic partial differential equation), $\alpha > 0$, and $\delta, \beta \geq 0$. The problem ($\mathbf{P}_{\alpha,\delta}$) can be interpreted as an inverse problem as well as an optimal control problem. Let us denote the solution with $u_{\alpha,\delta}$.

The estimate $\|u_{\alpha,0} - u_{\alpha,\delta}\|_{L^2(\Omega)} \leq \delta\,\alpha^{-1/2}$ for the error due to the noise level δ is well known for $\beta = 0$ and the proof can be extended to the case $\beta > 0$.

A typical way to estimate the regularization error as $\alpha \searrow 0$ is via a source condition, e.g. $u_{0,0} = \mathcal{S}^\star w$ with some $w \in H$. This yields $\|u_{\alpha,0} - u_{0,0}\|_{L^2(\Omega)} \leq C\,\alpha^{1/2}$. But if pointwise constraints are present ($U_{\text{ad}} = \{u \in L^2(\Omega) : u_a \leq u \leq u_b\}$), $u_{0,0}$ often is bang-bang, i.e. $u_{0,0}(x) \in \{u_a, 0, u_b\}$ a.e. in Ω. Hence, $u_{0,0} \notin H^1(\Omega)$ and by range$(\mathcal{S}^\star) \subset H^1(\Omega)$ a source condition with $\mathcal{S}^\star$ can not hold.

In this talk we present a new technique for deriving rates of the regularization error using a combination of a source condition and a regularity assumption on the adjoint variable $p_{0,0} = \mathcal{S}^\star(z_0 - \mathcal{S}u_{0,0})$. If the measure of $\big\{\big||p_{0,0}| - \beta\big| \leq \varepsilon\big\} \leq C\,\varepsilon$ for all $\varepsilon \geq 0$ it is possible to show $\|u_{\alpha,0} - u_{0,0}\|_{L^2(\Omega)} \leq C\,\alpha^{1/2}$ without a source condition.

We present examples showing that the error rates are sharp.

Inverse problems for Navier-Stokes equations

Masahiro Yamamoto

We consider an inverse problem of determining a spatially varying factor in a source term in a nonstationary linearized Navier-Stokes equations by observation data in an arbitrarily fixed sub-domain over some time interval. We prove the Lipschitz stability provided that the t-dependent factor satisfies a non-degeneracy condition. For the proof, we show a Carleman estimate for the Navier-Stokes equations.

This is a joint work with Mourad Choulli (University of Metz, France), Oleg Yu. Imanuvilov (Colorado State University, USA) and Jean-Pierre Puel (The University of Tokyo).

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