Chemnitz Symposium on Inverse Problems 2017 (on tour in Rio)

Adriano DeCezaro (FURG, Brazil) decezaromtm@gmail.com

On the Choice of the Tikhonov Regularization Parameter and the Discretization Level:
A Discrepancy-Based Strategy

Abstract:
We address the classical issue of appropriate choice of the regularization and discretization level for the Tikhonov regularization of an inverse problem with imperfectly measured data. We focus on the fact that the proper choice of the discretization level in the domain together with the regularization parameter is a key feature in adequate regularization. We propose a discrepancy-based choice for these quantities by applying a relaxed version of Morozov's discrepancy principle. Indeed, we prove the existence of the discretization level and the regularization parameter satisfying such discrepancy. We also prove associated regularizing properties concerning the Tikhonov minimizers.

Joint work with: Vinicius Albani, Jorge P. Zubelli

Alexandre Kawano (USP, Sao Paulo) ale.kawano@gmail.com

Simultaneous Identification of Source, Initial Conditions and Asynchronous Sources in the Vibration Problem of Euler-Bernoulli Beams.

Abstract:
In this talk we show under what conditions it is possible to uniquely identify simultaneously the source and initial conditions in a vibrating Euler-Bernoulli beam, when the available data is the observation of the displacement of a point during an arbitrary small interval of time. A counterexample is also shown to indicate that if some conditions are not satisfied then the unique identification is impossible.

Axel Osses (CMM, Chile) axosses@dim.uchile.cl

Algebraic reconstruction of source and attenuation in SPECT using first scattering measurements

Abstract:
Here we present an Algebraic Reconstruction Technique (ART) for solving the identification problem in Single Photon Emission Computed Tomography (SPECT). Ballistic and first scattering information are used to recover source and attenuation simultaneously. Both measurements are related with the Attenuated Radon Transform and a Klein-Nishina angular type dependency is considered for the scattering.

Joint work with: E.Cueva, M.Courdurier, J.-C.Quintana, C.Tejos, P.Irarrazaval

---

Barbara Kaltenbacher (Univ. Klagenfurt) Barbara.Kaltenbacher@aau.at

All-at-once and minimization based formulations of inverse problems and their regularization

Abstract:
The conventional way of formulating inverse problems such as identification of a (possibly infinite dimensional) parameter, is via some forward operator, which is the concatenation of the observation operator with the parameter-to-state-map for the underlying model. Recently, all-at-once formulations have been considered as an alternative to this reduced formulation, avoiding the use of a parameter-to-state map, which would sometimes lead to too restrictive conditions. Here the model and the observation are considered simultaneously as one large system with the state and the parameter as unknowns.
A still more general formulation of inverse problems, containing both the reduced and the all-at-once formulation, but also the well-known and highly versatile so-called variational approach (not to be mistaken with variational regularization) as special cases, is to formulate the inverse problem as a minimization problem (instead of an equation) for the state and parameter. Regularization can be incorporated via imposing constraints and/or adding regularization terms to the objective.

Bastian von Harrach (Univ. Frankfurt) harrach@math.uni-frankfurt.de

Monotonicity-based regularization of inverse coefficient problems

Abstract:
Newly emerging imaging methods lead to the inverse problem of determining one or several coefficient function(s) in an elliptic partial differential equation from (partial) knowledge of its solutions. A natural and generic approach is to approximate the unknown coefficients by minimizing a linearized and regularized data-fit functional. In this talk, we will show how to regularize the linearized data-fit functional with monotonicity-based constraints in such a way that convergence of certain shape properties in the reconstructions can be rigorously guaranteed.

Christian Clason (Univ. Duisburg-Essen) christian.clason@uni-due.de

Discrete regularization of parameter identification problems

Abstract:
This talk is concerned with parameter identification problems where a distributed parameter is known a priori to only take on values from a given discrete set. This property can be promoted in Tikhonov regularization with the aid of a suitable convex but nondifferentiable regularization term. This allows applying standard approaches to show well-posedness and convergence rates in Bregman distance. Using the specific properties of the regularization term, it can be shown that convergence (albeit without rates) actually holds pointwise. Furthermore, the resulting Tikhonov functional can be minimized efficiently using a semi-smooth Newton method. Numerical examples illustrate the properties of the regularization term and the numerical solution.

Christine Boeckmann (Univ. Potsdam) bockmann@rz.uni-potsdam.de

Modified Iterative Runge-Kutta-Type Methods for Nonlinear Ill-Posed Problems and Applications

Abstract:
This talk is devoted to the convergence analysis of a modified Runge-Kutta-type iterative regularization method for solving nonlinear ill-posed problems under a priori and a posteriori stopping rules. The convergence rate results of the proposed method can be obtained under a HÃ¶lder-type sourcewise condition if the FrÃ©chet derivative is properly scaled and locally Lipschitz continuous. Numerical results are achieved by using the Levenberg-Marquardt, Lobatto, and Radau methods. The Radau method yields proper results in Applications in Atmospheric Physics.

Joint work with: Pornsarp Pornsawad

Christopher Hofmann (TU Chemnitz) christopher.hofmann@mathematik.tu-chemnitz.de

Numerical studies of recovery chances for a simplified EIT problem

Abstract:
This study investigates a simplified discretized EIT model with eight electrodes distributed equally spaced at the boundary of a disk covered with a small number of material â€˜stripesâ€™ of varying conductivity. The goal of this paper is to evaluate the chances of identifying the conductivity values of each stripe from rotating measurements of potential differences. This setting comes from an engineering background, where the used EIT model is exploited for the detection of conductivities in carbon nanotubes (CNT) and carbon nanofibers (CNF). Connections between electrical conductivity and mechanical strain have been of major interest within the engineering community and has motivated the investigation of such a â€˜stripeâ€™ structure. Up to five conductivity values can be recovered from noisy 8 Ã— 8 data matrices in a stable manner by a least squares approach. Hence, this is a version of regularization by discretization and additional tools for stabilizing the recovery seem to be superfluous. To our astonishment, no local minima of the squared misfit functional were observed, which seems to indicate uniqueness of the recovery if the number of stripes is quite small.

Joint work with: Bernd Hofmann, Roman Unger

Fabio Margotti (UFSC, Brasil) fabiomarg@gmail.com

A projection strategy for choosing the regularization parameter of Iterated Tikhonov method in Banach spaces

Abstract:
We propose a strategy for choosing the sequence of regularization parameters of a variant in Banach spaces of the nonstationary Iterated Tikhonov (nIT) method. In this version, the penalization term of nIT is defined via the Bregman distance induced by a convex functional. The algorithm can be interpreted as a sequence of projections in some specific closed convex subsets. Under standard assumptions we prove strong convergence to a solution of the inverse problem in the noiseless case and the regularization property in case of noisy data.

Joint work with: Majela Machado, Antonio Leitao

Frank Werner (Max Planck Inst., Goetingen) Frank.Werner@mpibpc.mpg.de

Empirical Risk Minimization as Parameter Choice Rule for Filter based Regularization Methods

Abstract:
We consider a-posteriori parameter choice rules for filter based linear regularization methods in the statistical inverse problem setting.
In particular, we investigate the choice of the regularization parameter by minimizing an unbiased estimate of the predictive risk. This parameter choice rule and its usage are well-known in the literature, but oracle inequalities and optimality results in this general setting are unknown.

We prove a (generalized) oracle inequality, which relates the direct risk with the minimal prediction risk. From this oracle inequality, we are then able to conclude that the filter based regularization methods with the investigated parameter choice rule achieve the convergence rates of optimal order with respect to the mean integrated squared error in mildly ill-posed problems.

Finally, we also present numerical simulations, which support the order optimality of the method and the quality of the parameter choice. In these simulations, we also investigate the behavior of different a posteriori parameter choice methods in exponentially ill-posed problems.

Joint work with: Housen Li

Johann Baumeister (Frankfurt Univ.) baumeist@math.uni-frankfurt.de

Online Identification of parameters in time dependent differential equtions using partial observation

Abstract:
Mathematical models for a physical processes contain parameters which cannot be measured directly, in general. They must be determined by using recorded consequences they produce. Such problems are called inverse problems. Identification problems are inverse problems since one wants to reconstruct causes (parameter in a model) from consequences (output variables). Ill-posedness is a characteristic feature of inverse problems.
Online or real time parameter identification is the task of inferring model parameters simultaneously to the process of data sensing of a dynamical system in operation. Such techniques have been developed by engineers for control problems governed by ordinary differential equations. These techniques can also be used in the case of partial differential equations. We consider the model reference adaptive system-approach and develop a reference system which uses the observable part only. This study is inspired by a recent paper of Boiger and Kaltenbacher.

Michael Quellmalz (TU Chemnitz) michael.quellmalz@mathematik.tu-chemnitz.de

An SVD spherical surface wave tomography

Abstract:
The Funk-Radon transform, also known as the spherical Radon transform, assigns to a function on the sphere its mean values along all great circles.
Since its invention by Paul Funk in 1911, the Funk-Radon transform has been generalized to other families of circles as well as to higher dimensions.

We consider a generalization to circle arcs that is motivated by a problem from geomathematics. In spherical surface wave tomography, one measures the traveltimes of seismic surface waves on the Earth from an epicenter to a detector.
Then one wants to recover the phase velocity.
A common approach is the great circle ray approximation: we assume that a wave travels along a great circle and the traveltime equals the mean of the local phase velocity along the great circle arc connecting the epicenter and the receiver.
Mathematically, we want to recover the local phase velocity, which is a real-valued function defined on the sphere, from its integrals along certain arcs of great circles.
In this talk, we provide a singular value decomposition of the surface wave tomography with full data and for the case of great circle arcs whose length is fixed.

Joint work with: Ralf Hielscher, Daniel Potts

Peter Maass (Univ. Bremen) pmaass@math.uni-bremen.de

Machine learning for inverse problems

Abstract:
The classical approach to inverse problems starts with an analytical description $F:Xâ†’Y$ of the forward operator in some function spaces X,Y. The field of inverse problems addresses the task of reconstructing an unknown x^* from noisy data $y^\delta \sim F(x^*)$ with the further complication that F^{-1} or any type of generalized inverse is unbounded. The mathematical analysis stays within this framework and provides a regularization theory for optimal analytical convergence rates, stability estimates and convergence of numerical schemes.

This model driven approach has at least two shortcomings. First of all, the mathematical model is never complete. Extending the model might be challenging due to an only partial understanding of the underlying physical or technical setting. Secondly, most applications will have inputs which do not cover the full space X but stem from an unknown subset or obey an unknown stochastic distribution. E.g. there is no satisfactory mathematical model, which characterizes tomographic images or other image classes amongst all L_2-functions.

Machine learning offers several approaches for amending such analytical models by a data driven approach. Based on sets of training data either a specific problem adapted operator update is constructed and an established inversion process is used for regularizing the updated operator or the inverse problems is addressed by a machine learning method directly.

We present an overview on machine learning approaches for inverse problems. We include some first numerical experiments on how to apply deep learning concepts to inverse problems and we finsih by showing some real life applications of a model reduction/ basis learning approach in imaging mass spectrometry.

Robert Plato (Univ. Siegen) plato@mathematik.uni-siegen.de

Some new convergence rates results on variational Lavrentiev regularization for
nonlinear monotone ill-posed problems

Abstract:
In the present paper we consider the regularizing properties of the product midpoint rule for the stable solution of Abel-type integral equations of the first kind with perturbed right-hand sides. The impact of continuity and smoothness properties of solutions on the convergence rates is described in detailed manner by using a scale of H Ìˆ older spaces. In addition, correcting starting weights are introduced to get rid of undesirable initial conditions. The proof of the inverse stability of the quadrature weights relies on Banach algebra techniques. Finally, numerical results are presented.

In this talk we consider nonlinear ill-posed equations $ Fu=f $ in real Hilbert spaces $\mathcal{H}$, where the operator $F: \mathcal{H} \to \mathcal{H}$ is monotone on a closed convex subset $ \mathcal{M} \subset \mathcal{H}$, i.e.,

$$
\langle Fu-Fv, u-v \rangle \ge 0, \quad u, v \in \mathcal{M}.
$$

Since we admit perturbations $ f^\delta \in \mathcal{H}, \Vert f^\delta - f \Vert \le \delta $, some regularization of $ Fu=f $ is necessary. A standard approach is Lavrentiev regularization $Fv_\alpha^\delta + \alpha v_\alpha^\delta = f^\delta $, with a small regularization parameter $\alpha > 0$.
In many practical applications like parameter estimation problems in PDEs or the autoconvolution problem, the considered operator is indeed monotone on a closed convex subset $\mathcal{M} \subsetneq \mathcal{H} $ only. Since the Lavrentiev regularized equation may not have a solution $ v_\alpha^\delta \in \mathcal{M} $ under such general assumptions, we replace this equation by the corresponding regularized variational inequality and consider $u_\alpha^\delta \in \mathcal{M}$ satisfying

$$
\langle F u_\alpha^\delta + \alpha u_\alpha^\delta - f^\delta , w - u_\alpha^\delta \rangle
\ge 0 \quad \textup{for each } w \in \mathcal{M} .
$$

If a solution $u_* \in \mathcal{M}$ of the unperturbed equation $F u = f$ admits a source representation either of the form $ u_* = F'(u_*)z $ or $ u_* = F'(u_*)^* z $, with some $z \in \mathcal{H}$, where $ F'(u_*)^* $ denotes the adjoint operator of the Fr\'{e}chet derivative $ F'(u_*) $ of $ F $ at $ u_* $, then convergence rates for the error $\Vert u_\alpha^\delta - u_* \Vert$ are available for suitable a priori parameter choices $\alpha = \alpha(\delta)$. In this talk we present some new results in this direction.

Shuai Lu (Fudan Univ. Shanghai) slu@fudan.edu.cn

On parameter identification in linear stochastic differential equations by Gaussian statistics

Abstract:
Linear stochastic differential equations (SDE) arise in many contemporary sciences and engineering involving dynamical processes. These SDEs are governed by several parameters, for instance the damping coefficient, the volatility or diffusion coefficient and possibly an external forcing. Identification of these parameters allows a better understanding of the dynamical processes and its hidden statistics. By calculating the Gaussian statistics explicitly for the Ornstein--Uhlenbeck process with constant parameters and Langevin equations with periodic parameters, we propose a parameter identification approach recovering these parameters by minimizing the difference between the empirical statistics. The proposed approach is further extended to parameter identification of SDEs which is indirectly observed by another random variable.

Teresa Reginska (Polish Academy Warsaw) reginska@plusnet.pl

Adaptive discretization for the problem of identification of laser beam quality parameters

The problem of identification of laser beam quality parameters can be reduced to finding the waist of the axial profile defined by the radii of the beam. The regularized solutions of the Cauchy problem for the Helmholtz equation can be employed to approximate the axial profile at certain points. So we look for an approximate minimum of a function describing the axial profile given on a discrete set of points where its values are given with some errors. Presented is a new method for finding an approximate minimum of a real function f: The initial problem is replaced by that of finding parameters v âˆˆ R^d such that F(v,.) approximates f(.). Here F is appropriately chosen and F(v,.) are functions whose minima can easily be calculated. We introduce a modification of the iterative Tikhonov regularization in which the set of points (where noisy values of f are taken) changes at any step of iteration. The convergence of the method is proved but the rate of convergence is still an open problem.

Thomas Schuster (Univ. Saarbruecken) thomas.schuster@num.uni-sb.de

Thomographic terahertz imaging using sequential subspace optimization

Abstract:
Terahertz tomography aims for reconstructing the complex refractive index of a specimen, which is illuminated by electromagnetic radiation in the terahertz regime, from measurements of the resulting (total) electric field outside the object. The illuminating radiation is reflected, refracted, and absorbed by the object. In this work, we reconstruct the complex refractive index from tomographic measurements by means of regularization techniques in order to detect defects such as holes, cracks, and other inclusions, or to identify different materials and the moisture content. Mathematically, we are dealing with a nonlinear parameter identification problem for the two-dimensional Helmholtz equation, and solve it with the Landweber method and sequential subspace optimization. The article concludes with some numerical experiments.

Joint work with: Anne Wald

Uno Haemarik (Univ. Tartu, Estonia) Uno.Hamarik@ut.ee

Heuristic parameter choice in Tikhonov method from minimizers of the quasi-optimality function

Abstract:
We consider Tikhonov regularization for linear ill-posed operator equation in Hilbert spaces. In the case of the unknown noise level of the data the regularization parameter is chosen by some heuristic rule. From known heuristic parameter choice rules the best results were often obtained in the quasi-optimality criterion where the parameter is chosen as the global minimizer of the quasi-optimality function. For some problems this rule fails, the error of the Tikhonov approximation is very large. We prove that one of the local minimizers of the quasi-optimality function is always a good regularization parameter. We propose an algorithm for finding a proper local minimizer of the quasi-optimality function. Numerical examples for solving test problems of P. C. Hansen are given.

Joint work with: Toomas Raus

Urve Kangro (Univ. Tartu, Estonia) urve.kangro@ut.ee

On self-regularization of ill-posed problems in Banach spaces by projection methods

Abstract:
We consider ill-posed linear operator equations with operators acting between Banach spaces. For stable solution of ill-posed problems regularization is necessary and for using computers discretization is necessary. In some cases discretization may also be used as regularization method with discretization parameter as regularization parameter, additional regularization is not needed. Regularization by discretization is called self-regularization. We consider self-regularization by projection methods, giving necessary and sufficient conditions for self-regularization by a priori choice of the dimension of subspaces as the regularization parameter. Convergence conditions are also given for choice of the dimension by the discrepancy principle without requirement that the projection operators are uniformly bounded.

Joint work with: Uno Haemarik

Vinicius Albani (UFSC, Brazil) v.albani@ufsc.br

Optimal Convergence Rates Results for Linear Inverse Problems in Hilbert Spaces

Abstract:
We state optimal convergence-rate results for regularization methods in the solution of linear ill-posed operator equations in Hilbert spaces. They generalize some results in [3] to hold with general source conditions, including the logarithmic ones. In addition, the connection with approximative source conditions, introduced in [2], is provided. All these results are part of the recent work [1].

Joint work with: Peter Elbau, Mateen V. de Hoop, Otmar. Scherzer

References:
[1] V. Albani, P. Elbau, M. V. de Hoop and O. Scherzer
Optimal Convergence Rates Results for Linear Inverse Problems in Hilbert Spaces
Numer. Funct. Anal. Optim., (2016)

[2] B. Hofmann and P. MathÃ©
Analysis of Profile Functions for General Linear Regularization Methods
SIAM J. Numer. Anal., 45, 1122-1141 (2007)

[3] A. Neubauer
On converse and saturation results for Tikhonov regularization of linear ill-posed problems
SIAM J. Numer. Anal., 34, 517-527 (1997)

William Rundell (Texas T&M Univ.) rundell@math.tamu.edu

Inverse coefficient problems for diffusion operators: both classical and fractional order

Abstract:

Inverse problems for the heat equation or related parabolic equations are amongst the first of those studied. At the core of the often ill-conditioned problems is the diffusion process itself; it is difficult to reconstruct something that has undergone diffusion over time. This process, following Einstein's formulation of Brownian motion, indicates that the mean square expected value of a particle's position from its starting point is proportional to the evolved time. Yet over the last few decades this model has been challenged for certain materials -- leading to so-called anomalous diffusion processes. Many of these are based on fractional differential operators which are nonlocal and lead to a very different calculus. This in turn makes additional complexities in the diffusion operator. It also turns out, in some cases, to make serious changes to the inverse problem. We will highlight some "poster-children" for this effect and discuss some recent work for recovery of coefficients in both a fractional and a classical setting as a comparison.