CHEMNITZ UNIVERSITY OF TECHNOLOGY

Department of Mathematics
Analysis – Inverse Problems

Chemnitz Symposium on Inverse Problems 2011

Conference Guide

September 22 – 23, 2011

Chemnitz, Germany

General information
Timetable
Abstracts
List of participants

Imprint

Editor:
Prof. Dr. Bernd Hofmann
Chair of Analysis – Inverse Problems
Department of Mathematics
Chemnitz University of Technology

Editorial staff:
Jens Flemming

Office:
Reichenhainer Straße 39/41
09126 Chemnitz, Germany
Postal address:
09107 Chemnitz, Germany
Phone: +49 (0) 371 / 531-22000
Fax: +49 (0) 371 / 531-22009
Email: bernd.hofmann@mathematik.tu-chemnitz.de

ISSN 2190-7900

General information

Goal

Our symposium will bring together experts from the German and international 'Inverse Problems Community' and young scientists. The focus will be on ill-posedness phenomena, regularization theory and practice, and on the analytical, numerical, and stochastic treatment of applied inverse problems in natural sciences, engineering, and finance.

Location

Chemnitz University of Technology
Straße der Nationen 62 (Böttcher-Bau)
Conference hall 'Altes Heizhaus'
09111 Chemnitz, Germany

Selection of invited speakers

Sergei V. Pereverzyev (Linz, Austria)
Shuai Lu (Shanghai, China)
Markus Hegland (Canberra, Australia)
Paul E. Sacks (Ames, Iowa, USA)
Christine Böckmann (Potsdam, Germany)
Uno Hämarik (Tartu, Estonia)

Scientific board

Bernd Hofmann (Chemnitz, Germany)
Peter Mathé (Berlin, Germany)
Ulrich Tautenhahn (Zittau, Germany)

Organizing committee

Bernd Hofmann
Jens Flemming
Diana Roch
Nadja Rückert
Kerstin Seidel

Conference contacts

Prof. Dr. Bernd Hofmann
Jens Flemming

Chemnitz University of Technology
Department of Mathematics
Reichenhainer Straße 41
09126 Chemnitz, Germany

Email: ip2011@tu-chemnitz.de
Website: http://www.tu-chemnitz.de/mathematik/ip-symposium

Timetable

Overview for Thursday, September 22

09.00–09.05	**Opening**
09.05–10.15	**Session 1**
	M. Hegland, P. E. Sacks
10.15–10.35	**Coffee break**
10.35–11.45	**Session 2**
	C. Böckmann, U. Tautenhahn
11.45–13.00	**Lunch break**
13.00–14.25	**Session 3**
	J. Flemming, F. Werner, S. Becker
14.25–14.45	**Coffee break**
14.45–17.00	**Session 4**
	F. Schöpfer, T. Hein, J.-F. Pietschmann,
	N. Rückert, N. Worliczek
17.30	**Excursion**

Overview for Friday, September 23

09.00–09.10	**Laudation**
09.10–10.15	**Session 1**
	S. V. Pereverzyev, S. Lu
10.15–10.35	**Coffee break**
10.35–12.05	**Session 2**
	U. Hämarik, T. Regińska, R. Plato
12.05–12.50	**Lunch break**
12.50–15.15	**Session 3**
	M. Yamamoto, M. Gehre, A. Wöstehoff, V. Naumova,
	M. Yudytskiy, D. Gerth

Program for Thursday, September 22

09.00–09.05	**Opening**
09.05–09.40	**Markus Hegland** (Canberra, Australia) *Interpolation inequalities and regularisation theory*
09.40–10.15	**Paul E. Sacks** (Ames, Iowa, USA) *Inverse problems for the wave equation in an annulus*
10.15–10.35	**Coffee break**
10.35–11.10	**Christine Böckmann** (Potsdam, Germany) *Some inverse problems in aerosol physics*
11.10–11.45	**Ulrich Tautenhahn** (Zittau, Germany) *On implicit iteration methods for the stable solution of ill-posed problems*
11.45–13.00	**Lunch break**
13.00–13.30	**Jens Flemming** (Chemnitz, Germany) *Convergence rates for Poisson noise adapted Tikhonov regularization*
13.30–14.00	**Frank Werner** (Göttingen, Germany) *Iteratively regularized Newton methods for general data misfit functionals and applications to Poisson data*
14.00–14.25	**Saskia Becker** (Berlin, Germany) *Structural adaptive smoothing in* $\mathbb{R}^3 \rtimes \mathbb{S}^2$ *with application to diffusion weighted imaging*
14.25–14.45	**Coffee break**
14.45–15.15	**Frank Schöpfer** (Oldenburg, Germany) *Parameter identification problems in Banach spaces*
15.15–15.45	**Torsten Hein** (Berlin, Germany) *Iterative regularization schemes with general penalty*

15.45–16.10	**Jan-Frederik Pietschmann** (Cambridge, Great Britain) *Identification of non-linearities in transport-diffusion models of crowded motion*
16.10–16.35	**Nadja Rückert** (Chemnitz, Germany) *Regularization parameter choice methods for Poisson noise*
16.35–17.00	**Nadja Worliczek** (Braunschweig, Germany) *Tikhonov regularization with Bregman discrepancy and initial topologies*
	Excursion

Time periods include 5 minutes for discussion.

Program for Friday, September 23

The two morning sessions are dedicated to the 60$^{\text{th}}$ anniversary of Prof. Ulrich Tautenhahn.

09.00–09.10	**Bernd Hofmann** (Chemnitz, Germany) *Laudation*
09.10–09.45	**Sergei V. Pereverzyev** (Linz, Austria) *A meta-learning approach to the regularization – case study: blood glucose prediction*
09.45–10.15	**Shuai Lu** (Shanghai, China) *On the inverse problems for the coupled continuum pipe flow model for flows in karst aquifer*
10.15–10.35	**Coffee break**
10.35–11.05	**Uno Hämarik** (Tartu, Estonia) *A family of rules for choice of the regularization parameter in Tikhonov method for ill-posed problems with inexact noise*
11.05–11.35	**Teresa Regińska** (Warsaw, Poland) *A Cauchy problem for the Helmholtz equation on a rectangle*
11.35–12.05	**Robert Plato** (Siegen, Germany) *Some quadrature methods for perturbed weakly singular Volterra integral equations of the first kind*
12.05–12.50	**Lunch break**
12.50–13.20	**Masahiro Yamamoto** (Tokyo, Japan) *Coefficient inverse problem for transport equation*
13.20–13.45	**Matthias Gehre** (Bremen, Germany) *Sparsity constraint in electrical impedance tomography: complete electrode model*

13.45–14.10	**Arne Wöstehoff** (Hamburg, Germany) *Function space parameter estimation convergence in hyperelasticity*
14.10–14.35	**Valeriya Naumova** (Linz, Austria) *Adaptive parameter choice for one-sided finite difference schemes and its application in diabetes technology*
14.35–15.00	**Mykhaylo Yudytskiy** (Linz, Austria) *Wavelet-based methods in atmospheric tomography and adaptive optics*
15.00–15.15	**Daniel Gerth** (Chemnitz, Germany) *On an autoconvolution problem*

Time periods include 5 minutes for discussion.

Friday morning sessions dedicated to the 60th anniversary of Professor Ulrich Tautenhahn

January 12, 2011, the co-founder and member of the scientific board of the Chemnitz Symposium on Inverse Problems, Professor Ulrich Tautenhahn, has celebrated his 60th birthday. Since 1993 Ulrich Tautenhahn acts as a Professor of Mathematics at the Faculty of Mathematics and Natural Sciences of the University of Applied Sciences Zittau/Görlitz. In spite of the extreme teaching work load of 18 hours lectures per week he continuously publishes research papers in well-respected journals on inverse problems, regularization theory, ill-posed operator equations in Hilbert and Banach spaces, parameter identification in ODE and PDE models and on computational methods as well as algorithms for applied inverse problems.

Tautenhahn received his diploma degree in mathematics from the Technical University of Karl-Marx-Stadt in 1973 and his doctorate in mathematics in 1980. He was successful in various industrial projects on modeling, identification and prediction for different applied inverse problems. Between 1983 and 1985 he taught as an Assistant Professor at the University of Aleppo, Syrian Arabic Republic. In 1994, he defended his habilitation at the Chemnitz University of Technology.

Today, Professor Tautenhahn cultivates a close scientific cooperation with different experts of the inverse problems community from Germany, Austria, Estonia, Poland, India and other countries, frequently culminating in joint publications. He is a permanent reviewer for many international research journals. In 1997 he organized the 2nd Saxon Symposium on Inverse Problems - Mathematical

Methods and Applications, held in Oybin near Zittau. Ulrich Tautenhahn has held different research stays at the Johann Radon Institute for Computational and Applied Mathematics of the Austrian Academy of Sciences Linz, at the Weierstrass Institute for Applied Analysis and Stochastics Berlin and at the Institute of Mathematics of the Polish Academy of Sciences Warsaw. During his career, Tautenhahn has presented over 100 invited research lectures and conference talks in 15 foreign countries. He is married since 1981 and has two children.

For more information see:
http://www.hs-zigr.de/matnat/MATH/tautenhahn

Abstracts

Speaker	Page
Saskia Becker	12
Christine Böckmann	13
Jens Flemming	15
Matthias Gehre	16
Daniel Gerth	17
Uno Hämarik	18
Markus Hegland	19
Torsten Hein	20
Shuai Lu	22
Valeriya Naumova	23
Sergei V. Pereverzyev	24
Jan-Frederik Pietschmann	25
Robert Plato	26
Teresa Regińska	28
Nadja Rückert	30
Paul E. Sacks	31
Frank Schöpfer	32
Ulrich Tautenhahn	33
Frank Werner	35
Nadja Worliczek	36
Arne Wöstehoff	38
Masahiro Yamamoto	39
Mykhaylo Yudytskiy	40

Structural adaptive smoothing in $\mathbb{R}^3 \rtimes \mathbb{S}^2$ with application to diffusion weighted imaging

Saskia Becker, Jörg Polzehl, Karsten Tabelow,
Henning U. Voss

The data generating process of diffusion weighted magnetic resonance imaging (DW-MRI) can be expressed as $\mathbb{S}^2$-convolution of the fiber orientation density function (ODF) with a response function which describes the profile of a typical fiber bundle in the human brain, spinal cord, or muscle tissue [Tournier et al., 2004]. Hence, the fiber ODF can be obtained by spherical deconvolution. As all imaging modalities DW-MRI suffers from significant random noise why noise reduction is essential.

In this talk we will present a smoothing method for DW-MRI data $S : \mathbb{R}^3 \rtimes \mathbb{S}^2 \to \mathbb{R}$ using the embedding of $\mathbb{R}^3 \rtimes \mathbb{S}^2$ into $SE(3)$ as introduced by [Franken, 2008]. The algorithm is based on the Propagation-Separation (PS) approach [Polzehl and Spokoiny, 2006] and can be applied without reference to a specific model like the diffusion tensor model or any HARDI modelling. Further, this algorithm is able to reduce noise without blurring the structural borders between areas with constant parameter. We exhibit properties of this structural adaptive method, both for simulated and experimental data.

Some inverse ill-posed problems in aerosol physics

Christine Böckmann, Lukas Osterloh

Retrieving the distribution of aerosols in the atmosphere via remote sensing techniques is a highly complex task that requires dealing with a wide range of different problems stemming both from physics and mathematics. In this talk, we focus on retrieving this distribution from multi-wavelength Lidar (optical Radar) data for aerosol ensembles consisting of spherical and spheroidal particles.

The optical efficiencies for spherical particles, which form the kernel function of the resulting integral equation, play a very crucial part here. Their behavior with respect to the refractive index of the particles and other properties is examined, also leading to an investigation into the degree of ill-posedness of the problem. The nature of log-normal distributions and their retrieval is analyzed, as they are often assumed for aerosols. From these results, improved algorithms based on a Padé iteration for the retrieval of the particle distribution for spherical particles can be constructed and tested for their accuracy. The influence of the common assumption of wavelength independence of the refractive index is tested for its validity. For the retrieval of the complex refractive index, a Monte-Carlo type algorithm is proposed, which results in a speedup without loss of accuracy when compared to the commonly used fixed-grid approach.

The optical efficiencies for spheroidal particles, and the differences to those for spherical ones, are investigated. For that purpose, T matrix code is used for the calculation of spheroidal efficiencies. The influence of particle shape on depolarization, which is zero for spheres, is analyzed. To consider the fact that ensembles of spheroidal particles will have particles of different aspect ratios, a two-dimensional forward model is introduced that allows two-dimensional particle distributions, depending on both the size and the shape of the particles. The algorithms developed previously for spheres are successfully adapted to suit the inversion with variable aspect ratio. Simulations with several different ensembles of spheroidal particles show that the availability of depolarization information is crucial to obtain information on the particle shape in the inversion.

To validate the results, the developed algorithms are applied to different cases of measurement data gained from multi-wavelength Lidar devices. In some of the cases, the results are successfully validated against other sources.

Convergence rates for Poisson noise adapted Tikhonov regularization

Jens Flemming

We consider operator equations where the right-hand side is some energy density. According to this density photons are emitted and the available information about the right-hand side is the number of photons observed by an array of detectors in a fixed time interval. Such photon counts typically follow a Poisson distribution. For large counts the Gaussian distribution yields a good approximation, but for low photon counts we have to work with the Poisson distribution.

Applications where low photon counts appear are for instance positron emission tomography (PET), confocal laser scanning microscopy (CLSM), and image processing in astronomy.

Choosing a suitable fitting functional the Tikhonov regularization method allows to handle ill-posed operator equations based on Poisson distributed data. We present basic results on well-posedness and we show how to extend the technique of variational inequalities introduced in [1] to obtain convergence rates for the Poisson noise adapted Tikhonov method. Further, we proof that a source condition yields the desired variational inequality even in the case of this non-standard method.

Numerical examples illustrate the influence of the choice of the fitting functional on the regularized solutions.

References

[1] B. Hofmann, B. Kaltenbacher, C. Pöschl, O. Scherzer: *A convergence rates result for Tikhonov regularization in Banach spaces with non-smooth operators.* Inverse Problems, 2007, vol. 23, 987–1010.

Sparsity constraint in electrical impedance tomography: complete electrode model

Matthias Gehre, Peter Maaß, Bangti Jin

We consider the complete electrode model for Electrical Impedance Tomography, which is more sophisticated than the Neumann-to-Dirichlet map. It matches experimental data to measurement precision. We proceed by showing some of its analytical properties using the $W^{1,p}$ regularity of the associated second order elliptic PDE. Those results allow us to justify a reconstruction approach based on minimizing a Tikhonov functional with an l_1-penalty term. Finally we present some promising reconstructions from experimental data.

On an autoconvolution problem

Daniel Gerth

The autoconvolution equation has been examined intensively in the context of inverse problems. However, results for complex valued functions occurring as solutions and right-hand sides of the autoconvolution equation and for nontrivial kernels are missing in the literature. Motivated by a new method of measuring ultra-short laser pulses developed at the Max-Born-Institute for nonlinear optics and short-time spectroscopy, Berlin, there were serious reasons to bridge this gap. The talk will present results of a Diploma Thesis prepared and supervised in Berlin and Chemnitz, where analytical and numerical results for that autoconvolution problem could be achieved.

A family of rules for choice of the regularization parameter in Tikhonov method for ill-posed problems with inexact noise level

Uno Hämarik, Reimo Palm, and Toomas Raus

We consider the equation $Au = f, f \in \mathcal{R}(A)$, where A is linear bounded operator acting between two Hilbert spaces and only an approximation $\tilde{f} \in F$ with inexact noise level δ is given instead of f. Here δ may be a serious overestimation with $\|\tilde{f} - f\| \leq \delta$, but also underestimation of the noise level is possible : for example, it may be known only that with high probability $\delta/\|y - y_*\| \in [1/10, 10]$. We consider choice of the regularization parameter α in the Tikhonov method $u_\alpha = (\alpha I + A^* A)^{-1} A^* \tilde{f}$.

As says Bakushinski's veto, the convergence of u_α can be guaranteed only if the choice of α uses the noise level. Classical rules for parameter choice as the discrepancy principle, monotone error rule and the balancing principle are unstable with respect to the inaccuracies of the noise level: they fail in case of underestimated noise level and give large error of u_α already at very moderate overestimation of the noise level. We propose for choice of $\alpha = \alpha(\delta)$ the following family of rules which includes all known quasi-optimal rules.

Define $B_\alpha = \sqrt{\alpha}(\alpha I + AA^*)^{-1/2}$, $D_\alpha = \alpha^{-1} AA^* B_\alpha^2$. Fix the parameters q, k, l such that $3/2 \leq q < \infty$, $l \geq 0$, $k \geq l/q$. Choose $\alpha = \alpha(\delta)$ as the largest solution of the equation

$$\frac{(1 + \alpha\|A\|^{-2})^{((k+s_0)q-l)/(2q-2)} \|D_\alpha^k B_\alpha (Au_\alpha - \tilde{f})\|^{q/(q-1)}}{\|D_\alpha^l B_\alpha^{2q-2} (Au_\alpha - \tilde{f})\|^{1/(q-1)}} = b\delta,$$

where b is constant large enough and $s_0 = 0$ if $k = l/q$, $s_0 = 1/2$ if $k > l/q$.

We analyze the quasi-optimality and the stability of these rules. The advantages of some rules of this family over classical rules in case of the over- or underestimated noise level with $\delta/\|\tilde{f} - f\| \in [1/10, 10]$ are demonstrated on extensive numerical experiments in test problems of P.C. Hansen.

Interpolation inequalities and regularisation theory

Markus Hegland

Interpolation between spaces is used in analysis to define new spaces and analyse operators. Furthermore, interpolation is an important tool in approximation theory in the investigation of approximation classes. In regularisation theory, interpolation inequalities are the main tool to get error bounds which relate source conditions and data errors to errors in the solutions.

In this talk interpolation will be reviewed from a regularisation theory perspective and several known and new error bounds and results based on interpolation inequaities will be discussed.

Iterative regularization schemes with general penalty

Torsten Hein

We consider the linear ill-posed operator equation

$$A\,x = y \qquad x \in X,\ y \in Y,$$

where $A : X \longrightarrow Y$ denotes a linear bounded operator mapping between the Banach spaces X and Y. For $\delta > 0$ and given noisy data $y^\delta \in Y$ with knowing bound $\|y^\delta - y\| \le \delta$ for the noise level we deal with the iteration approach

$$\begin{aligned} x_0^\delta &= x_0 := G(x_0^*),\ x_0^* \in X^* \\ x_{n+1}^* &:= x_n^* - \mu_n \phi_n^* \\ x_{n+1}^\delta &:= G(x_{n+1}^*) \end{aligned}$$

together with the discrepancy principle as stopping criterion for the iteration process. Here, for smooth spaces Y, $\phi_n^* \in X^*$ denotes the gradient of the objective functional $x \mapsto \frac{1}{p}\|A\,x - y^\delta\|^p$ at the element x_n^δ. We deal with the following questions:

- How should we choose the operator $G : X^* \longrightarrow X$ in order to support specific properties of the regularized solutions such as sparsity and discontinuities?
- How should we determine the step size μ_n in order to obtain a tolerable speed of convergence of this gradient-type algorithm?
- Under which conditions does regularization really hold?

Moreover, quite similar iteration schemes occur in literature for constraint (finite dimensional) minimization such as (linearized) Bregman methods as well as for minimization approaches for Tikhonov functionals with L^1/l^1 or TV penalty term. We discuss how these approaches are related to our scheme which somehow demonstrate the numerical advantage of iterative regularization.

The presentation will be closed by a short numerical example.

On the inverse problems for the coupled continuum pipe flow model for flows in karst aquifer

Bang Hu, Nan Chen, Jin Cheng, Shuai Lu

We investigate several inverse problems of the the coupled continuum pipe flow (CCPF) model which describes the fluid flows in karst aquifer. We focus on the case in which the exchange rate is a space-dependent variable. After generalizing the well-posedness of the forward problem to the anisotropic exchange rate case, we present the uniqueness of this parameter by the Cauchy data. Besides, the uniqueness of the geometry of the conduit by the Cauchy data is verified as well. These results enhance the practicality of the CCPF model.

Adaptive parameter choice for one-sided finite difference schemes and its application in diabetes technology

Valeriya Naumova, Sergei V. Pereverzev,
Sivananthan Sampath

In this paper we discuss the problem of an adaptive parameter choice in one-sided finite difference schemes for the numerical differentiation in case when noisy values of the function to be differentiated are available only at the given points. This problem is motivated by diabetes therapy management, where it is important to provide estimations of the future blood glucose concentration from current and past measurements. Here we show, how the proposed approach can be used for this purpose and demonstrate some illustrative tests, as well as the results of numerical experiments with simulated clinical data.

A meta-learning approach to the regularization – case study: blood glucose prediction

Sergei Pereverzev, Sivananthan Sampath

We are going to present a new scheme of a regularization, where the regularization space and the regularization parameter are adaptively chosen within the regularization procedure. The construction of such Fully Adaptive Regularization/Learning algorithm (FARL) is motivated by the problem of predicting the blood glucose concentration of diabetic patients. We describe how proposed scheme can be used for this purpose and report the results of numerical experiments with real clinical data. The presentation is based on the joint patent application with Sivananthan Sampath (RICAM), Jette Randlov (Novo Nordisk A/S) and Samuel McKennoch (Novo Nordisk A/S) filed on April, 20, 2011.

Identification of non-linearities in transport-diffusion models of crowded motion

Martin Burger, Jan-Frederik Pietschmann,
Marie-Therese Wolfram

The aim of this talk is to formulate a class of inverse problems of particular relevance in crowded motion, namely the simultaneous identification of entropies and mobilities. We study a model case of this class, namely the identification from flux-based measurements in a stationary setup. This leads to an inverse problem for a non-linear transportation diffusion model, where boundary values and possibly an external potential can be varied. In specific settings we provide a detailed theory for the forward map and an adjoint problem useful in the analysis and numerical solution. We further verify the simultaneous identifiability of the non-linearities and present several numerical tests yielding further insight on the way variations in boundary values and external potential affect the quality of reconstructions.

Some quadrature methods for perturbed weakly singular Volterra integral equations of the first kind

Robert Plato

The subject of this talk is the regularization of linear weakly singular Volterra integral equations of the following form,

$$\frac{1}{\Gamma(\alpha)}\int_0^x (x-y)^{\alpha-1}k(x,y)u(y)dy = f(x), \qquad 0 \le x \le x_{\max}, \quad (1)$$

with $0 < \alpha < 1$ and $x_{\max} > 0$, and Γ denotes Euler's gamma function. It is assumed that the kernel function k is sufficiently smooth and satisfies $k(x,x) = 1$ for $0 \le x \le x_{\max}$. The function $f : [0, x_{\max}] \to \mathbb{R}$ is supposed to be approximately given, and a function $u : [0, x_{\max}] \to \mathbb{R}$ satisfying equation (1) has to be determined.

Quadrature methods for the approximate solution of equation (1) are well studied if the right-hand side f is exactly given. The composite midpoint rule is one of these methods, and it is considered, for exact data, e. g. in [1]. In the present talk we consider this method for perturbed right-hand sides in equation (1). More precisely, we assume that approximations $f_n^\delta \in \mathbb{R}$ with $|f_n^\delta - f(x_n)| \le \delta$ are available for $n = 1, 2, \dots, N$, where $x_n = nh$, $n = 1, 2, \dots, N$ are uniformly distributed grid points, with $h = x_{\max}/N$, and N is a positive integer. In addition, $\delta > 0$ is a given noise level. In this situation an application of the composite midpoint rule leads to the following linear system of equations:

$$\frac{h^\alpha}{\Gamma(\alpha+1)}\sum_{j=1}^{n}((n+1)^\alpha - n^\alpha)k(x_n, x_j)u_j^\delta = f_n^\delta, \quad n = 1, 2, \dots, N. \quad (2)$$

We consider the regularizing properties of method (2) and present some numerical results. Similar results for the composite trapezoidal method and fractional multistep methods are also included in the presentation.

References

[1] P. P. B. Eggermont. Special discretization methods for the integral equations of image reconstruction and for Abel-type integral equations. *Technical Report, University of New York*, Buffalo 1981.

A Cauchy problem for the Helmholtz equation on a rectangle

Teresa Regińska

A new approach to a reconstruction of the radiation field from experimental data given on a part of boundary will be presented. The model problem for a collimated laser beam is described by a Cauchy problem for the Helmholtz equation on a rectangle with measurement data accessible on one its side only:

$$\begin{cases} \Delta u + k^2 u = 0, & \text{in } \Omega = (0,a) \times (0,b) \\ u(x,0) = g(x), & x \in (0,a), \\ u_y(x,0) = h(x), & x \in (0,a). \end{cases}$$

It is assumed that g, h are such that the solution exists in H^2. It is known that if u exists, then it is unique.

With respect to real experiments, it is reasonable to assume that measurement data are given only on the side of rectangle most distant from the sources. This is the main difference between our approach and previous ones, where additional homogeneous or periodic boundary conditions are taken into account on the sides parallel to the beam axis, i.e. for $x = 0$, and $x = a$. However, the homogeneous boundary conditions have no clear physical meaning, and periodic boundary conditions can be applied only in case of symmetric beams.

Therefore, standard transformation into a problem in frequency space cannot be directly applied, because the related Fourier series are not termwise differentiable (as it is in the case of homogeneous boundary conditions on the sides parallel to the beam axis). Using the idea proposed by A.G. Ramm, we reduce a part of nonhomogeneous boundary values to the homogeneous case. A series representation of the solution is derived with infinite system of differential equations describing their coefficients.

This series representation of the solution can be a starting point to formulation of different regularization methods. One example of possible regularization techniques based on this representation will be presented: a spectral type regularization method. The obtained error bound depends on the regularization parameter, measurement error and a priori bounds for certain norms of the unknown solution traces on the boundary.

Regularization parameter choice methods for Poisson noise

Nadja Rückert

In some applications, e.g. astronomy, medical applications, images are only available as numbers of photons detected at each pixel. These photon counts are modeled as a Poisson process and therefore imply the generalized Kullback-Leibler divergence as the fitting functional. The aim is to recover the original image from the Poisson distributed data via a Tikhonov-type functional. In this talk, some methods for choosing the regularization parameter are presented.

Inverse problems for the wave equation in an annulus

Paul Sacks

Let $u(x,t)$ denote the solution of

$$u_{tt} - \Delta u - q(x)u = \delta(x,t) \qquad x \in \mathbb{R}^3 \quad t > 0$$

$$u(x,0) = u_t(x,0) = 0 \qquad x \in \mathbb{R}^3$$

We will discuss some uniqueness results for the problem of recovering q in the annulus $A := \{x \in \mathbb{R}^3 : R_1 < |x| < R_2\}$ given Cauchy data on either the inner or outer surface of A.

Parameter identification problems in Banach spaces

Frank Schöpfer

In this talk we are concerned with nonlinear parameter identification problems that can be cast as nonlinear operator equations $F(x) = y$, where $F : D \subset X \to Y$ is a nonlinear operator between Banach spaces X, Y and only noisy data y^δ is available with known noise level $\|y - y^\delta\| \leq \delta$. We discuss convergence and regularizing properties of an accelerated Landweber type iteration. Acceleration is achieved by Bregman projections onto intersections of stripes whose width depends on the noise level and the nonlinearity of F expressed by the tangential cone condition. The efficiency of the method is illustrated by some numerical examples.

On implicit iteration methods for the stable solution of ill-posed problems

Ulrich Tautenhahn

In this talk we consider ill-posed problems $Ax = y$ where A is a linear, injective and bounded operator between Hilbert spaces X and Y with non-closed range $\mathcal{R}(A)$. We assume that $y \in \mathcal{R}(A)$ so that the operator equation $Ax = y$ has a unique solution $x^\dagger \in X$. We further assume that y is unknown and that $y^\delta \in Y$ is the available right hand side with $\|y - y^\delta\| \leq \delta$. Problems of this kind have to be regularized. In this paper we study *implicit iteration methods in Hilbert scales*, in which regularized solutions x_n^δ are obtained by

$$\begin{aligned} x_k^\delta &= x_{k-1}^\delta - (A^*A + \alpha_k B^{2s})^{-1} A^*(Ax_{k-1}^\delta - y^\delta), \\ &\quad k = 1, 2, ..., n, \\ x_0^\delta &= x_0 \end{aligned} \tag{1}$$

where $B : \mathcal{D}(B) \subset X \to X$ is some unbounded densely defined self-adjoint strictly positive definite operator, $\alpha_k > 0$ are properly chosen real numbers, s is some generally nonnegative number that controls the smoothness to be introduced into the regularization procedure and x_0 is some properly chosen starting value. In these regularization methods, the positive number

$$\sigma_n := \sum_{k=1}^{n} \frac{1}{\alpha_k} \tag{2}$$

plays the role of the regularization parameter. In the talk we review on known results and add some new aspects:

- We report on order optimal error bounds (on general source sets) for $\|x_n^\delta - x^\dagger\|$ in case σ_n is chosen either *a priori* or *a posteriori*, e.g., by the discrepancy principle.

- We discuss the question of choosing a (new) efficient starting value α_1 in the iteration (1) that allows a considerable reduction of the number n of necessary iteration steps.

- For realizing *a posteriori* rules of choosing σ_n, that is, for choosing n and $(\alpha_k)_{k=1}^{n}$ in (2), fast algorithms are proposed which are based on Newton's method applied to some properly transformed equations.

- One of our main results is that in our proposed special methods only two or three iteration steps in (1) are required.

Iteratively regularized Newton methods with general data misfit functionals and applications to Poisson data

Thorsten Hohage, Frank Werner

In this talk we present results on Newton type methods for inverse problems which are described by nonlinear operator equations

$$F(u) = g, \qquad u \in \mathfrak{B}, g \in \mathcal{Y}.$$

The Newton equations $F(u_n) + F'(u_n; u_{n+1} - u_n) = g$ are regularized variationally using a general convex data misfit functional and a convex regularization term:

$$u_{n+1} := \operatorname*{argmin}_{u \in \mathfrak{B}} \left[\mathcal{S}\left(F(u_n) + F'(u_n; u - u_n); g^{\mathrm{obs}} \right) + \alpha_n \mathcal{R}(u) \right].$$

This leads to a generalization of the well-known iteratively regularized Gauss-Newton method (IRGNM) and is of interest for example in photonic imaging, where one observes Poisson data and the Kullback-Leibler divergence is considered as natural data misfit functional.

We present convergence rates under generalized source conditions in form of variational inequalities and a generalized tangential cone condition. We also mention some recent results on rates of convergence for Hölder-type source conditions with index $\nu \in \left(\frac{1}{2}, 1\right)$ under a Lipschitz-type nonlinearity condition.

The performance of the described method is illustrated by numerical results for a phase retrieval problem.

References

[1] T. Hohage and F. Werner. *Iteratively regularized Newton methods with general data misfit functionals and applications to Poisson data.* http://arxiv.org/abs/1105.2690, 2011

Tikhonov regularization with Bregman discrepancy and initial topologies

Dirk Lorenz, Nadja Worliczek

In recent years Tikhonov regularization with general discrepancy terms, i.e. regularizing the solution of operator equations $Ku = v$ by minimizing functionals

$$T_{\alpha,v}(u) = \mathcal{S}(Ku, v) + \alpha R(u)$$

with a distance like functional $\mathcal{S}$ became a matter of growing interest, since it allows to incorporate a variety of noise models.

As pointed out in [1], a crucial point for standard variational proof techniques as in [2] and [3] are certain properties of the chosen topologies on the data space V, i.e. the relation between convergence of a sequence $v_n \to v$ and $\mathcal{S}(v, v_n) \to 0$ and sequential continuity of $\mathcal{S}(v, \cdot\,)$ on its domain of definition.

For exploring which discrepancy functionals $\mathcal{S}$ meet the above requirements for a given topological setting, we propose to study the relation of the given topologies to the initial topology induced by $(\mathcal{S}(v, \cdot\,))_{v \in V}$ on the respective domains of definition.

In this talk we will discuss this for the special case of $\mathcal{S}$ being a Bregman distance. Moreover we will dwell on regularization properties of such Tikhonov functionals with respect to the initial topology itself.

References

[1] Jens Flemming. *Theory and examples of variational regularization with non-metric fitting functionals.* J. Inverse Ill-Posed Probl., 18(6): 677–699, 2010.

[2] Bernd Hofmann, Barbara Kaltenbacher, Christiane Pöschl, and Otmar Scherzer. *A convergence rates result for Tikhonov regulariza-*

tion in Banach spaces with non-smooth operators. Inverse Problems, 23(3):987–1010, 2007.

[3] Christiane Pöschl. *Tikhonov Regularization with General Residual Term.* PhD thesis, Leopold Franzens Universität Innsbruck, Innsbruck, Austria, October 2008.

Function space parameter estimation convergence in hyperelasticity

Arne Wöstehoff

Localization of hidden damages using ultrasonic waves is a popular field in structural health monitoring. The material undergoes a dynamic input and certain components of the displacement field are measured at different points. Conclusions are drawn from these measurements about locations of cracks, delaminations and other damages.

If the linear hyperelastic model of continuum mechanics

$$\rho\ddot{u} = L^{\top} C L u \tag{$*$}$$

is used to describe anisotropic wave propagation, we are aimed to identify the elasticity tensor C, which is a function of the position. To this end the functional

$$J(C) := \|Qu(C) - \xi\|^2 + \beta\|C - C_0\|^2$$

should be minimized over an admissible set of elasticity tensors, subject to $u(C)$ fulfilling $(*)$ and proper initial and boundary conditions.

This infinite-dimensional optimization problem is approximated by a series of finite-dimensional ones in the sense of function space parameter estimation convergence, presented in detail in [1].

We concentrate on the identification of an adequate elasticity tensor space and the forward problem's unique solvability. We provide a preview on the approximation problem.

References

[1] H. T. Banks and K. Kunisch, *Estimation Techniques for Distributed Parameter Systems*, Birkhäuser, Boston, 1989.

Coefficient inverse problem for transport equation

Manabu Machida, Masahiro Yamamoto

We consider wave or particles propagation in a random medium. Then the density of particle flow $u(x, v, t)$ at time t and position x with the velocity v satisfies a radiative transport equation with the absorption coefficient $\sigma_a(x, v)$ and the scattering coefficient $\sigma_s(x, v)$. We discuss an inverse problem of determining σ_a and/or σ_s by extra data of u on subset of the lateral boundary. By a Carleman estimate, we prove the Lipschitz stability estimate for the inverse problem. Moreover we show that our stability estimate for the inverse problem is the best possible.

Wavelet-based methods in atmospheric tomography and adaptive optics

Mykhaylo Yudytskiy

The problem of atmospheric tomography arises in ground-based telescope imaging with adaptive optics, where one aims to compensate in real-time for the rapidly changing optical distortions in the atmosphere. The mathematical formulation of the problem resembles limited angle tomography. The recent developments of wavelet-based reconstructions methods in atmospheric tomography are discussed. In this talk we give a short introduction to the topic, discuss the theoretical results, and present a few numerical examples.

List of participants

Saskia Becker
Weierstrass Institue for Applied Analysis and Stochastics (WIAS)
Berlin, Germany
saskia.becker@wias-berlin.de
http://www.wias-berlin.de/∼beckers

Ismael Bleyer
Johannes Kepler University Linz,
Doctoral Program 'Computational Mathematics'
Linz, Austria
ismael.bleyer@dk-compmath.jku.at
https://www.dk-compmath.jku.at/people/ibleyer

Christine Böckmann
University of Potsdam, Institute of Mathematics
Potsdam, Germany
christine.boeckmann@uni-potsdam.de
http://www.math.uni-potsdam.de/∼bockmann

Jens Flemming
Chemnitz University of Technology, Department of Mathematics
Chemnitz, Germany
jens.flemming@mathematik.tu-chemnitz.de
http://www.tu-chemnitz.de/∼jgei

Matthias Gehre
University of Bremen, Center for Industrial Mathematics
Bremen, Germany
mgehre@math.uni-bremen.de
http://www.math.uni-bremen.de/zetem/matthiasgehre

Daniel Gerth
Chemnitz University of Technology, Department of Mathematics
Chemnitz, Germany
daniel.gerth@s2005.tu-chemnitz.de

Rudolf Gorenflo
Freie Universität Berlin,
Department of Mathematics and Computer Science
Berlin, Germany
gorenflo@mi.fu-berlin.de
http://www.math.fu-berlin.de/profs/gorenflo.html

Uno Hämarik
University of Tartu, Faculty of Mathematics and Informatics
Tartu, Estonia
uno.hamarik@ut.ee
http://www.math.ut.ee/325002

Sybille Handrock-Meyer
Chemnitz University of Technology, Department of Mathematics
Chemnitz, Germany
handrock@mathematik.tu-chemnitz.de
http://www.tu-chemnitz.de/∼syha

Markus Hegland
Australian National University, Mathematical Sciences Institute
Canberra, Australia
markus.hegland@anu.edu.au
http://maths.anu.edu.au/∼hegland

Torsten Hein
Technische Universität Berlin, Institute of Mathematics
Berlin, Germany
hein@math.tu-berlin.de
http://www.tu-chemnitz.de/∼thein

Bernd Hofmann
Chemnitz University of Technology, Department of Mathematics
Chemnitz, Germany
bernd.hofmann@mathematik.tu-chemnitz.de
http://www.tu-chemnitz.de/mathematik/inverse_probleme/hofmann

Shuai Lu
Fudan University, School of Mathematical Sciences
Shanghai, China
slu@fudan.edu.cn
http://homepage.fudan.edu.cn/shuailu

Valeriya Naumova
Johann Radon Institute for Computational and Applied Mathematics (RICAM)
Linz, Austria
valeriya.naumova@oeaw.ac.at
http://www.ricam.oeaw.ac.at/people/page.cgi?firstn=Valeriya;lastn=Naumova

Sergei V. Pereverzyev
Johann Radon Institute for Computational and Applied Mathematics (RICAM)
Linz, Austria
sergei.pereverzyev@oeaw.ac.at
http://www.ricam.oeaw.ac.at/people/page.cgi?firstn=Sergei;lastn=Pereverzyev

Jan-Frederik Pietschmann
University of Cambridge,
Department of Applied Mathematics and Theoretical Physics
Cambridge, Great Britain
jan.pietschmann@googlemail.com
http://www.jfpietschmann.eu

Robert Plato
University of Siegen, Department of Mathematics
Siegen, Germany
plato@mathematik.uni-siegen.de
http://www.uni-siegen.de/fb6/aan/plato

Teresa Regińska
Institute of Mathematics of the Polish Academy of Sciences
Warsaw, Poland
reginska@impan.pl
http://www.impan.pl/∼reginska

Diana Roch
Chemnitz University of Technology, Department of Mathematics
Chemnitz, Germany
diana.roch@mathematik.tu-chemnitz.de
http://www.tu-chemnitz.de/∼droc

Arnd Rösch
University of Duisburg–Essen, Department of Mathematics
Duisburg, Germany
arnd.roesch@uni-due.de
http://www.uni-due.de/mathematik/agroesch/roesch

Nadja Rückert
Chemnitz University of Technology, Department of Mathematics
Chemnitz, Germany
nadja.rueckert@mathematik.tu-chemnitz.de
http://www.tu-chemnitz.de/∼ngo

Paul E. Sacks
Iowa State University, Department of Mathematics
Ames, Iowa, USA
psacks@iastate.edu
http://www.public.iastate.edu/∼psacks

Frank Schöpfer
University of Oldenburg, Institute of Mathematics
Oldenburg, Germany
frank.schoepfer@uni-oldenburg.de
http://www.uni-oldenburg.de/index/personen/?username=FSchoepfer

Yuanyuan Shao
University of Applied Sciences Zittau/Goerlitz,
Faculty of Mathematics and Natural Sciences
Zittau, Germany
Chemnitz University of Technology, Department of Mathematics
Chemnitz, Germany
yshao@hs-zigr.de

Ulrich Tautenhahn
University of Applied Sciences Zittau/Goerlitz,
Faculty of Mathematics and Natural Sciences
Zittau, Germany
u.tautenhahn@hs-zigr.de
http://www.hs-zigr.de/matnat/MATH/tautenhahn

Ivan Tomba
University of Bologna, Department of Mathematics
Bologna, Italy
ivan.tomba3@unibo.it

Dana Uhlig
Chemnitz University of Technology, Department of Mathematics
Chemnitz, Germany
dana.uhlig@mathematik.tu-chemnitz.de
http://ww.tu-chemnitz.de/∼dana

Gerd Wachsmuth
Chemnitz University of Technology, Department of Mathematics
Chemnitz, Germany
gerd.wachsmuth@mathematik.tu-chemnitz.de
http://www.tu-chemnitz.de/mathematik/part_dgl/people/wachsmuth/index.en.php

Frank Werner
Georg-August-Universität Göttingen,
Institute for Numerical and Applied Mathematics
Göttingen, Germany
f.werner@math.uni-goettingen.de
http://num.math.uni-goettingen.de/f.werner

Nadja Worliczek
Technische Universität Braunschweig,
Institute for Analysis and Algebra
Braunschweig, Germany
n.worliczek@tu-bs.de
http://www.tu-braunschweig.de/iaa/personal/worliczek

Arne Wöstehoff
Helmut Schmidt University Hamburg
Hamburg, Germany
arne.woestehoff@hsu-hh.de
http://www.hsu-hh.de/mb-mathe/index_bm1UbY58WQeMWQXk.html

Masahiro Yamamoto
University of Tokyo, Department of Mathematical Sciences
Tokyo, Japan
myama@ms.u-tokyo.ac.jp

Mykhaylo Yudytskiy
Johann Radon Institute for Computational and Applied
Mathematics (RICAM)
Linz, Austria
mykhaylo.yudytskiy@ricam.oeaw.ac.at
http://people.ricam.oeaw.ac.at/m.yudytskiy

ISSN 2190-7900