Brownian motion and the Feynman-Kac formula on Riemannian manifolds, TU Chemnitz, WS 2021/2022,

Prof. Dr. Batu Güneysu,

Exercise sheet 7

1. Let U_n, $n \in \mathbb{N}$, be an exhaustion of M by open subsets. Prove that for all $0 \leq f \in L^2(M)$, $t > 0$, one has

$$P_t^{U_n} f|_{U_n} \nearrow P_t f \quad \mu\text{-a.e.}$$

2. Let $\zeta : I \times M \to \mathbb{R}$ be continuous (with some interval $I \subset \mathbb{R}$), and assume $\partial_t \zeta$ exists (classically) and is continuous on $I \times M$. Show that for all open relatively compact $U \subset M$, the Banach-space valued map

$$I \ni t \mapsto \zeta(t, \cdot) \in C_b(U)$$

is strongly differentiable, and that its strong derivative $(d/dt)\zeta$ equals $\partial_t \zeta$ on $I \times U$.