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CHEMNITZ

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Simulations involving the multivariate normal distribution

Perspective: *Computerpraktikum*
Writing hints: https://www.overleaf.com/read/hdpkgxgjkbgw

Preliminaries

Suppose the random vector $X \in \mathbb{R}^d$ follows a multivariate normal distribution with mean μ and covariance matrix Σ,

$$X \sim \mathcal{N}(\mu, \Sigma), \tag{1}$$

where $\mu \in \mathbb{R}^d$ and $\Sigma \in \mathbb{R}^{d\times d}$ is positive definite. Samples from X can be generated by $X := \mu + L(Z_1, \dots, Z_d)^\top$, where $Z_i \sim \mathcal{N}(0,1)$, $i = 1, \dots, d$, are independent, each following a standard normal distribution and $L \cdot L^\top = \Sigma$. For $X \sim \mathcal{N}(\mu, \Sigma)$, it holds that

$$\mathbb{E}\, e^{t^\top X} = e^{\mu^\top t + \frac{1}{2} t^\top \Sigma t}, \tag{2}$$

where $t \in \mathbb{R}^d$. References include Liptser and Shiryaev [1, 2], cf. also https://www.tu-chemnitz.de/mathematik/fima/public/LecturesAndMaterials/mathematischeStatistik.pdf.

Task

1. Implement numerical realizations of the multivariate distribution (1).
2. For some $t \in \mathbb{R}^d$ fixed, plot histograms of

$$Y_t := e^{t^\top X}$$

   and verify the result (2) empirically for varying choices of $t \in \mathbb{R}^d$.
3. Summarize the results and findings in a brief report.
4. The result (2) holds for $t \in \mathbb{C}^d$ as well. Repeat 2 by studying the real and imaginary part separately.

References

[1] R. S. Liptser and A. N. Shiryaev. *Statistics of Random Processes I*. Springer, 2nd edition, 2001. doi:10.1007/978-3-662-13043-8. 1

[2] R. S. Liptser and A. N. Shiryaev. *Statistics of Random Processes II*. 2nd edition, 2001. doi:10.1007/978-3-662-10028-8. 1