UNIVERSITY OF TECHNOLOGY
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CHEMNITZ

Faculty of Mathematics
Mathematical Finance
Alois Pichler

Portfoliooptimization

Perspective: *Seminar thesis, Bachelor, Master's thesis*
Writing hints: https://www.overleaf.com/read/hdpkgxgjkbgw

Preliminaries

In what follows, $\xi \in \mathbb{R}^d$ is a vector of random returns, for example the return of d stocks. A fraction x_i of the total budget (1 EUR) is invested in stock i, $i = 1, \dots, d$, so that the total (random) return of the portfolio is

$$\sum_{i=1}^{d} x_i \cdot \xi_i = x^\top \xi.$$

Let X be a random variable and $\mathcal{R}$ a *risk measure*, for example[1]

$$\mathcal{R}(X) := \mathsf{AV@R}_\alpha(X) = \inf_{q \in \mathbb{R}} q + \frac{1}{1-\alpha} \mathbb{E}(X-q)_+,$$

the *average value-at-risk*.

Problem formulations

For some random return ξ, the following are different portfolio optimization problems.

1. Maximize the profit by controlling the risk by some threshold $q \in \mathbb{R}$,

$$\begin{aligned} \text{maximize}_{(\text{in } x \in \mathbb{R}^d)} \quad & \mathbb{E}\, x^\top \xi \\ \text{subject to} \quad & -\mathcal{R}(-x^\top \xi) \geq q, \\ & x^\top \mathbb{1} = \sum_{i=1}^{n} x_i = 1 \text{ EUR} \qquad \text{(budget constraint)}, \\ & x_i \geq 0,\ i = 1, \dots, d \qquad \text{(no shortselling)}. \end{aligned}$$

[1] $x_+ := \max(0, x)$

2. Minimize the risk while requiring a minimal average return $\mu \in \mathbb{R}$,

$$\begin{aligned} \text{minimize } & \mathcal{R}(-x^\top \xi) \\ \text{subject to } & \mathbb{E}(x^\top \xi) \geq \mu, \\ & x^\top \mathbb{1} = \sum_{i=1}^n x_i = 1 \text{ EUR}, \\ & x_i \geq 0,\ i = 1, \dots, d. \end{aligned}$$

3. Integrated risk management, with $\tilde{\mathcal{R}}(X) \coloneqq (1-\lambda)\,\mathbb{E}\,X + \lambda \cdot \mathcal{R}(X)$ and $\lambda \in (0,1)$,

$$\begin{aligned} \text{maximize } & -\tilde{\mathcal{R}}(-x^\top \xi) = (1-\lambda) \cdot \mathbb{E}\, x^\top \xi - \lambda \cdot \mathcal{R}(-x^\top \xi) \\ \text{subject to } & x^\top \mathbb{1} = \sum_{i=1}^n x_i = 1 \text{ EUR and} \\ & x_i \geq 0,\ i = 1, \dots, d. \end{aligned}$$

For details, cf. also https://www.tu-chemnitz.de/mathematik/fima/public/LecturesAndMaterials/PortfolioOptimierung.pdf.

Task

For the return ξ of your favorite d assets,

1. implement the optimization problems above;
2. discuss and compare the results for varying parameters q, μ and λ or risk measures;
3. plot the optimal investments in adequate charts.

Bonne chance!