Pension fund ALM with Multivariate Second order Stochastic Dominance constraints

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Purpose of the work

To model and implement an Asset Liability Management problem of a Pension Fund in a Defined Benefit framework having:
- a short-term profitability target,
- a medium-term insurance risk-adjusted return
- a long-term strategic objective

Definition of multivariate second order stochastic dominance between the wealth of the Pension Fund and a benchmark wealth

The multivariate Second order Stochastic Dominance (SSD) is formulated with three alternatives which we investigate
The multivariate SSD
Let \((\Omega, F, P)\) denote a probability space and let \(X\) and \(Y\) be two random variables having as cumulative distribution functions \(F_X\) and \(F_Y\).

Let’s define the **twice cumulative distribution function** as

\[
F_X^{(2)}(\eta) = \int_{-\infty}^{\eta} F_X(\alpha) d\alpha
\]

We say that \(X\) dominates \(Y\) in the Second order Stochastic Dominance (SSD) sense, \(X >_{SSD} Y\), if

\[
F_X^{(2)}(\eta) \leq F_Y^{(2)}(\eta), \quad \forall \eta \in \mathbb{R}
\]

If the random variables are **discrete** and then represented by random vectors \(X\) and \(Y\), and if the realizations are **equiprobable**, then the SSD is equivalent to

\[
X \leq WY
\]

where \(W\) is a double stochastic matrix.
When we consider **multivariate** random variables, we need to re-think the SSD relation.

Assume that a multivariate random variable $\mathbf{X}$ has $T$ dimensions and then we observe $X_t, t = 1, \ldots, T$ univariate random variables.

If the random variable is **discrete**, each univariate random variable $X_t$ can be represented with with a vector $\mathbf{x}_t$, then $\mathbf{X}$ can be represented with a matrix $\mathbf{X}$ having $T$ columns, one for each dimension:

$$
\mathbf{X} = 
\begin{bmatrix}
  x_{1,1} & \ldots & x_{1,T} \\
  \vdots & \ddots & \vdots \\
  x_{S,1} & \ldots & x_{S,T}
\end{bmatrix}
$$

The meaning of $\mathbf{X} \succ_{SSD} \mathbf{Y}$ is not unique and can be declined in various ways. We analyze three of them.
Component-wise Multivariate SSD (C-MSSD)

\[ X \succ_{SSD} Y \iff X_t \succ_{SSD_t} Y_t \quad \forall t \text{ (disjointly)} \]

\[ \succ_{SSD_1} \quad \succ_{SSD_2} \quad \succ_{SSD_3} \]
Linear Multivariate SSD (Lin-MSSD)

\[ X \gtrsim^{\text{lin}}_{\text{SSD}} Y \iff \sum_{t=1}^{T} \bar{X}_t \cdot c_t \gtrsim_{\text{SSD}} \sum_{t=1}^{T} c_t \sum_{t=1}^{T} Y_t, \forall c_t \geq 0 \]
MultiDimension Multivariate SSD (MD-MSSD)

\[ X \succ_{SSD} Y \iff X_t \succ_{SSD} Y_t \quad \forall t \text{ (jointly)} \]
Multivariate SSD

The three possible definitions:

- **The Component-wise Multivariate SSD (C-MSSD):**
  \[ X \succ_{SSD} Y \ 	ext{iff} \ X_t \succ_{SSD_t} Y_t \ orall t \text{ (disjointly)} \]
  \[ X_t \leq W_t Y_t, \ orall t \]

- **The Linear Multivariate SSD (Lin-MSSD):**
  Dencheva and Ruszczynski (2009), Dentcheva and Wolfhagen (2015, 2016)
  \[ X \succ_{SSD}^{\text{lin}} Y \ 	ext{iff} \ \sum_{t=1}^{T} c_t X_t \succ_{SSD} \sum_{t=1}^{T} c_t Y_t, \ orall c_t \geq 0 | \sum_{t=1}^{T} c_t = 1 \]
  \[ c^T X \leq W(c) c^T Y, \ orall c \geq 0, \sum_{t=1}^{T} c_t = 1 \]

- **The MultiDimension Multivariate SSD (MD-MSSD):**
  \[ X \succ_{SSD} Y \ 	ext{iff} \ X_t \succ_{SSD} Y_t \ orall t \text{ (jointly)} \]
  \[ X_t \leq W Y_t, \ orall t \]
Multivariate SSD

The three possible definitions:

• The Component-wise Multivariate SSD (C-MSSD):
  \[ X \succ_{SSD} Y \iff X_t \succ_{SSD} Y_t \quad \forall t \text{ (disjointly)} \]
  \[ X_t \leq W_t Y_t, \quad \forall t \]

• The Linear Multivariate SSD (Lin-MSSD):
  Dencheva and Ruszczynski (2009), Dentcheva and Wolffhagen (2015, 2016)
  \[ X \succ_{SSD}^\text{lin} Y \iff \sum_{t=1}^T c_t X_t \succ_{SSD} \sum_{t=1}^T c_t Y_t, \quad \forall c_t \geq 0 \mid \sum_{t=1}^T c_t = 1 \]
  \[ c^T X \leq W c^T Y, \quad \forall c \geq 0, \quad \sum_{t=1}^T c_t = 1 \]

• The MultiDimension Multivariate SSD (MD-MSSD):
  \[ X \succ_{SSD} Y \iff X_t \succ_{SSD} Y_t \quad \forall t \text{ (jointly)} \]
  \[ X_t \leq W Y_t, \quad \forall t \]
The ALM model
Approach structure

- **Financial Datafeed**
  - Portfolio Universe
  - Risk factors

- **Simulation input**
  - Econometric model definition
  - Econometric model estimation
  - Population model setting
  - Stochastic tree structure

- **Monte Carlo simulator**
  - Nodal financial coefficient generation
  - Monte Carlo scenario generation
  - Population actuarial simulation

- **Stochastic Programming**
  - Dynamic portfolio model
  - Stochastic program solution

- **Solution analysis**
## Extended Asset Universe

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Asset List</th>
<th>Lower &amp; Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>Cash</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Floaters</td>
<td>0%</td>
</tr>
<tr>
<td>Treasuries</td>
<td>Treasury 1-3y</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Treasury 3-5y</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Treasury 5-7y</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Treasury 7-10y</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Treasury 10+y</td>
<td>0%</td>
</tr>
<tr>
<td>Securitized</td>
<td>Securitized</td>
<td>0%</td>
</tr>
<tr>
<td>Corporate</td>
<td>Corporate Inv Grade</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Corporate High Yield</td>
<td>0%</td>
</tr>
<tr>
<td>Public Equity</td>
<td>Public Equity</td>
<td>0%</td>
</tr>
<tr>
<td>Real Estate</td>
<td>Real Estate</td>
<td>0%</td>
</tr>
</tbody>
</table>
Notation for sets

- Time partition from time 0 to year 20
  \[ T = \{ t_0 = 0, 1, 2, ..., H \} \]

- Set of decision times
  \[ T_d = \{ t_0 = 0, 1, 2, 3, 5, 10, H \} \]

- Set of intermediate stages
  \[ T_{int} = T \setminus \{ T_d \} = \{ 4, 6, 7, 8, 9, 11, ..., 19 \} \]
Notation for investment variables

Buying decision in stage $t$, scenario $s$, of asset $i$

\[ x_{i,t,s}^+ \]

Selling in stage $t$, scenario $s$, of asset $i$ that was bought in $h$

\[ x_{i,h,t,s}^- \]

Expiry of a fixed-income asset in stage $t$, scenario $s$, of asset $i$ that was bought in $h$

\[ x_{i,h,t,s}^{\text{exp}} \]

Holding in stage $t$, scenario $s$, of asset $i$ that was bought in $h$

\[ x_{i,h,t,s} \]

Cash account in stage $t$, scenario $s$

\[ z_{t,s} = z_{t,s}^+ - z_{t,s}^- \]

Sponsors’ unexpected contributions

\[ \Phi_{t,s}^k \]
Variable definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net pension payments</td>
<td>$L_{t,s}^{NET}$</td>
</tr>
<tr>
<td>Defined benefit obligation (DBO)</td>
<td>$\Lambda_{t,s}$</td>
</tr>
<tr>
<td>Asset value</td>
<td>$X_{i,t,s} = \sum_{h &lt; t_j} x_{i,h,t_j,s}$</td>
</tr>
<tr>
<td>Asset portfolio value</td>
<td>$CX_{t_j,s} = \sum_{i \in I} X_{i,t_j,s} + z_{t_j,s}^{+}$</td>
</tr>
<tr>
<td>Net Defined benefit obligation</td>
<td>$B_{t_j,s} = \Lambda_{t_j,s} - CX_{t_j,s}$</td>
</tr>
<tr>
<td>Intermediate net payments</td>
<td>$L_{t_j,s}^{Z} = \sum_{h &lt; t_j, h \in T} x_{i,h,t_j-1} \cdot \xi_{i,t_j,s} +$</td>
</tr>
<tr>
<td></td>
<td>$\sum_{h &lt; t_j, t_j-h \geq T_i} x_{i,h,t_j,s}^{exp} - L_{t,j,s}^{NET}$</td>
</tr>
</tbody>
</table>
Variable definitions

Liquidity gap plus ALM risk

\[ \Psi_{t,j,s} = \Omega_{t,j,s} + K^1_{t,j,s} + \Psi_{t_{j-1},s}, \quad \Psi_{t,0,s} = 0 \]

Liquidity gap

\[ \Omega_{t,j,s} = L_{t,j,s}^{NET} - \sum_{t_{j-1} < h < t_j} L^Z_{h,s} (1 + \zeta_{t,s}) \]

ALM risk

\[ K^1_{t,j,s} = dr^+ \cdot (t_j - t_{j-1}) \cdot (\Delta^x_{t,j,s} - \Delta^A_{t,j,s})^+ \]
\[ - dr^- \cdot (t_j - t_{j-1}) \cdot (\Delta^x_{t,j,s} - \Delta^A_{t,j,s})^- \]
### Variable definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realized portfolio return</td>
<td>( \Pi_{t,j,s}^{INV} ) ( = \Pi_{t,j,s}^{1,INV} + G_{t,j,s} )</td>
</tr>
<tr>
<td>Coupon return</td>
<td>...</td>
</tr>
<tr>
<td>Capital gain return</td>
<td>...</td>
</tr>
<tr>
<td>Total portfolio return</td>
<td>( \Pi_{t,j,s} = \Pi_{t,j,s}^{INV} + UGL_{t,j,s} - UGL_{t,0,s} )</td>
</tr>
<tr>
<td>Unrealized gain and losses</td>
<td>( UGL_{t,j,s} = \sum_{i \in I} \sum_{h &lt; t_j, h \in T} \chi_{i,h,t,j,s} \cdot \chi_{i,h,t,j,s} )</td>
</tr>
<tr>
<td>Cumulated realized portfolio return</td>
<td>( \Pi_{t,s}^{INV,cum} = \sum_{t_k \leq t_j} \Pi_{t,k,s}^{INV} )</td>
</tr>
<tr>
<td>Cumulated total portfolio return</td>
<td>( \Pi_{t,s}^{cum} = \Pi_{t,s}^{INV,cum} + UGL_{t,j,s} - UGL_{t,0,s} )</td>
</tr>
</tbody>
</table>
Variable definitions

Total risk capital

\[ K_{t,j,s} = K_{t,j,s}^{TEC} + K_{t,j,s}^{INV} \]

Actuarial risk capital

\[ K_{t,j,s}^{TEC} = \phi \cdot \Lambda_{t,s} \]

Investment risk capital

\[ K_{t,j,s}^{INV} = K_{t,j,s}^{1} + K_{t,j,s}^{M} + K_{t,j-1,s}^{INV} \]

Market risk

\[ K_{t,j,s}^{M} = \sum_{n=2,...,12} \sum_{h<t_j,h\in T_d} (x_{n,h,t_j,s}) \cdot k_n \cdot (t_j - t_{j-1}) \]

ALM risk

\[ K_{t,j,s}^{1} = dr^+ \cdot (t_j - t_{j-1}) \cdot (\Delta x_{t,j,s} - \Delta^\Lambda_{t,j,s})^+ \]
\[ - dr^- \cdot (t_j - t_{j-1}) \cdot (\Delta x_{t,j,s} - \Delta^\Lambda_{t,j,s})^- \]
**Variable definitions**

- **Total portfolio return per unit tail risk**
  
  \[ Z_{t,j,s} = \frac{\Pi_{t,j,s}^{cum}}{K_{t,j,s}^{INV} + \Phi_{t,j,s}^k} \]

- **Total extraordinary plan sponsors’ contributions**
  
  \[ \Phi_{t,j,s} = \Phi_{t,j,s}^k + \Phi_{t,j-1,s} \]
Other constraints

- Inventory balance constraints at time $t_0 = 0$, root node
- Inventory balance constraints at time $t_j$
- Cash balance constraints at time $t_0 = 0$, root node
- Cash balance constraints at time $t_j$
- Single asset upper bound
- Single asset lower bound
- Asset class upper bound
- Asset class lower bound
- Turnover constraint
- Liquidity constraint
Objective formulation

Objective function:

$\text{MAX} \quad \text{Expected Value} \quad \text{MIN} \quad \text{Expected Shortfall}$

$$\left(1 - \alpha\right) \sum \lambda_j \mathbb{E}[Y_{j,t}] - \alpha \sum \lambda_j \mathbb{E}[Y_j - Y_{j,t}|Y_{j,t} < \bar{Y}_j]$$

- **Short-term profitability**
  - Liquidity Gap + ALM Risk
  - H&N: 10%

- **Industrial plan target**
  - RORAC
  - 30%

- **Long-term sustainability**
  - Sponsor Injection
  - Net DBO
  - 20%
Dynamic Asset Allocation

Existent Portfolio

- Real Estate: 10%
- Public Equity: 25%
- Corporates: 13%
- Securitized: 6%
- Treasuries: 31%
- Cash: 15%

H&N Optimal Solution

- Real Estate: 10%
- Public Equity: 36%
- Corporates: 11%
- Securitized: 6%
- Treasuries: 6%
- Cash: 30%

Cutting-edge stochastic optimization framework

Benchmark portfolio

Dynamic Optimal Solution
## Results – Benchmark

<table>
<thead>
<tr>
<th>Actual Alloc.</th>
<th>SSD 7</th>
<th>C-MSSD 6-7</th>
<th>C-MSSD 5-7</th>
<th>MD-MSSD 6-7</th>
<th>MD-MSSD 5-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% 25% 13% 6% 31% 15%</td>
<td>10% 5% 13% 20% 23% 30%</td>
<td>10% 9% 19% 6% 48% 7%</td>
<td>10% 11% 15% 7% 57% 7%</td>
<td>10% 9% 19% 6% 48% 57%</td>
<td>10% 11% 15% 6% 48% 57%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std Dev</th>
<th>V@R</th>
<th>AV@R</th>
<th>Obj Val</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>242,783</td>
<td>248,066</td>
<td>551,419</td>
<td>1,014,506</td>
<td>551,426</td>
<td>1,014,435</td>
</tr>
<tr>
<td>239,930</td>
<td>243,017</td>
<td>441,119</td>
<td>690,723</td>
<td>441,025</td>
<td>690,490</td>
</tr>
<tr>
<td>4,675</td>
<td>9,190</td>
<td>120,375</td>
<td>314,693</td>
<td>120,373</td>
<td>314,620</td>
</tr>
<tr>
<td>555</td>
<td>1,644</td>
<td>79,387</td>
<td>254,560</td>
<td>79,455</td>
<td>254,689</td>
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<tr>
<td>-31,950</td>
<td>-31,973</td>
<td>-59,747</td>
<td>-123,889</td>
<td>-59.748</td>
<td>-123,890</td>
</tr>
<tr>
<td>166</td>
<td>232</td>
<td>421</td>
<td>335</td>
<td>1,622</td>
<td>1,956</td>
</tr>
</tbody>
</table>
### Results – Benchmark 2

<table>
<thead>
<tr>
<th>Actual Alloc.</th>
<th>SSD</th>
<th>C-MSSD 6-7</th>
<th>C-MSSD 5-7</th>
<th>MD-MSSD 6-7</th>
<th>MD-MSSD 5-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>242,783</td>
<td>496,286</td>
<td>1,310,899</td>
<td>2,398,575</td>
<td>1,310,899</td>
</tr>
<tr>
<td>Std Dev</td>
<td>239,930</td>
<td>545,782</td>
<td>897,851</td>
<td>1,412,045</td>
<td>897,851</td>
</tr>
<tr>
<td>V@R</td>
<td>4,675</td>
<td>53,442</td>
<td>377,758</td>
<td>867,990</td>
<td>377,758</td>
</tr>
<tr>
<td>AV@R</td>
<td>555</td>
<td>27,055</td>
<td>302,775</td>
<td>760,485</td>
<td>302,775</td>
</tr>
<tr>
<td>Time</td>
<td>166</td>
<td>771</td>
<td>859</td>
<td>650</td>
<td>2391</td>
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<tr>
<td>SSD</td>
<td>C-MSSD 6-7</td>
<td>C-MSSD 5-7</td>
<td>MD-MSSD 6-7</td>
<td>MD-MSSD 5-7</td>
<td></td>
</tr>
<tr>
<td>--------</td>
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<td>-------------</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3,673,643</td>
<td>2,059,912</td>
<td>3,673,642</td>
<td>2,084,574</td>
<td></td>
</tr>
<tr>
<td>6-7</td>
<td>2,059,915</td>
<td>1,329,444</td>
<td>2,059,912</td>
<td>1,346,306</td>
<td></td>
</tr>
<tr>
<td>5-7</td>
<td>1,346,306</td>
<td>644,790</td>
<td>1,346,306</td>
<td>1,111,726</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1,329,444</td>
<td>644,790</td>
<td>1,329,444</td>
<td>1,111,726</td>
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<td>7</td>
<td>2,084,574</td>
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<td>1,111,726</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Actual Alloc.</th>
<th>SSD</th>
<th>C-MSSD</th>
<th>C-MSSD</th>
<th>MD-MSSD</th>
<th>MD-MSSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% Real Estate</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>25% Public Equity</td>
<td>5%</td>
<td>19%</td>
<td>25%</td>
<td>19%</td>
<td>25%</td>
</tr>
<tr>
<td>13% Corporates</td>
<td>13%</td>
<td>13%</td>
<td>13%</td>
<td>13%</td>
<td>13%</td>
</tr>
<tr>
<td>6% Securitized</td>
<td>23%</td>
<td>6%</td>
<td>6%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>31% Treasuries</td>
<td>30%</td>
<td>53%</td>
<td>48%</td>
<td>53%</td>
<td>48%</td>
</tr>
<tr>
<td>15% Cash</td>
<td>41%</td>
<td>53%</td>
<td>48%</td>
<td>53%</td>
<td>48%</td>
</tr>
</tbody>
</table>

- **Mean:**
  - SSD: 242,783
  - C-MSSD: 745,815
  - C-MSSD: 3,673,643
  - MD-MSSD: 2,059,912
  - MD-MSSD: 3,673,642

- **Std Dev:**
  - SSD: 239,930
  - C-MSSD: 848,462
  - C-MSSD: 2,084,549
  - MD-MSSD: 1,329,444
  - MD-MSSD: 2,084,574

- **V@R:**
  - SSD: 4,675
  - C-MSSD: 95,059
  - C-MSSD: 1,346,306
  - MD-MSSD: 644,790
  - MD-MSSD: 1,111,726

- **AV@R:**
  - SSD: 555
  - C-MSSD: 55,136
  - C-MSSD: 485,869
  - MD-MSSD: 485,882
  - MD-MSSD: 1,111,726

- **Obj Val:**
  - SSD: -31,950
  - C-MSSD: -72,798
  - C-MSSD: -623,047
  - MD-MSSD: -283,463
  - MD-MSSD: -623,047

- **Time:**
  - SSD: 166
  - C-MSSD: 1,416
  - C-MSSD: 387
  - MD-MSSD: 1097
  - MD-MSSD: 1380
<table>
<thead>
<tr>
<th>Real Estate</th>
<th>Public Equity</th>
<th>Corporates</th>
<th>Securitized</th>
<th>Treasuries</th>
<th>Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark</strong></td>
<td></td>
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</tr>
<tr>
<td>SSD</td>
<td>C</td>
<td>C</td>
<td>MD</td>
<td>MD</td>
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<tr>
<td>7</td>
<td>6-7</td>
<td>5-7</td>
<td>6-7</td>
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<tr>
<td><strong>Benchmark 2</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>SSD</td>
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<td>MD</td>
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<tr>
<td><strong>Benchmark 3</strong></td>
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<tr>
<td>SSD</td>
<td>C</td>
<td>C</td>
<td>MD</td>
<td>MD</td>
<td></td>
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### Summary Table

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<th>Benchmark</th>
<th>Benchmark 2</th>
<th>Benchmark 3</th>
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</tr>
</tbody>
</table>

- **Benchmark:** SSD 7, C 6-7, C 5-7, MD 6-7, MD 5-7
- **Benchmark 2:** SSD 7, C 6-7, C 5-7, MD 6-7, MD 5-7
- **Benchmark 3:** SSD 7, C 6-7, C 5-7, MD 6-7, MD 5-7
Conclusions

The alternative versions of the Multivariate SSD are very close to each other.

The MD-MSSD is a stronger condition and its meaning is more clear and reasonable.

The MD-MSSD is more computational demanding, but still tractable.


Bibliography – Stochastic Dominance


Thank you