Mixed Finite Volume Element Methods for Three-field Formulations in Elasticity and Poroelasticity

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We propose a family of mixed finite element and finite volume element methods for the approximation of linear elastostatics, formulated in terms of displacement, rotation vector, and solid pressure. We discuss the unique solvability of the continuous three-field formulation, as well as the invertibility and stability of the proposed Galerkin and Petrov-Galerkin formulations. Optimal a priori error estimates are derived using norms that are robust with respect to the Lamé constants, turning these numerical methods to be particularly appealing for nearly incompressible materials. The predicted accuracy and applicability of the new three-field formulation and the corresponding mixed finite (volume) element schemes is verified numerically by conducting a number of computational tests in both 2D and 3D.

Furthermore, we introduce a second order finite volume element formulation for a stationary three-field poroelasticity problem on a 2D domain, where we approximate the solid displacement, the pore-pressure of the fluid and an auxiliary unknown representing the volumetric part of the total stress. The well-posedness of the proposed finite volume element method can only be established provided that suitable mesh geometric requirements are imposed on the family mesh partitions and it is also shown that the optimal error estimates are robust with respect to Lamé's first parameter approaching infinity, i.e. when the locking phenomenon occurs.

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