On the Accuracy and Robustness of A Posteriori Error Majorants for Approximate Solutions of Reaction-Diffusion Equations

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Dedicated to Ulrich Langer on his 65th birthday.

Efficiency of the error control at numerical solutions of partial differential equations entirely depends on two factors: accuracy of an a posteriori error majorant and the computational cost of its evaluation, including the cost of a testing function or a vector field employed for the evaluation. In the paper, consistency of an a posteriori error bound implies that it is the same in the order with the respective unimprovable a priori bound and, therefore, it is a basic characteristic related to the first factor.

The paper is dedicated to the elliptic reaction-diffusion equations, which are modelled by the equation $-\Delta u + \sigma u = f$ in Ω and $u = 0$ on the boundary $\partial\Omega$. We present a guaranteed robust a posteriori error majorant which is effective at any constant reaction coefficient $\sigma \geq 0$. Additionally, for the FEM (finite element method) solutions the majorant is consistent under assumptions that the mesh is quasiuniform and senior coefficients of the equation and the domain are sufficiently smooth. For big values of σ the majorant coincides with the majorant of Aubin (1972), the accuracy of which, as it is well known, deteriorates at σ tending to zero while at $\sigma = 0$ the majorant loses its sense. For $\sigma \in [0, \sigma_*]$ we upgrade Aubin's majorant , where for FEM solutions $\sigma_* = ch^2$ and $c = c(\partial\Omega) = const$. In fact, we prove that for such σ the multiplier $1/\sigma$, which in Aubin's majorant stands before the square of L^2-norm of the residual type term, can be replaced by ch^2 with the same constant as above. Similar correction is applicable to a number of other known a posteriori error majorants with the residual type terms in the right parts. In particular it is applicable to the majorants for approximate solutions of the Poisson and reaction-diffusion equations which do not lose their sense at $\sigma = 0$, but nevertheless are not consistent with the a priori error bounds.

The a posteriori error majorant developed in the paper can be expanded on a wider range of problems. In this relation we mention elliptic equations with piece wise constant nonnegative σ and elliptic equations of the 2n-th order, $n \geq 1$. A part of the results is presented in the papers of Korneev (2016-2017) refered below. Research was supported by the grant from the Russian Fund of Basic Research, project N 15-01-08847à.

References:

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