Relating FEM to FVM for Interface Problems in CFD

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We consider multiphase flow problems such as particle-fluid and gas-fluid mixtures to derive suitable finite element spaces for the Navier-Stokes equations. There is still ongoing research on the suitable choice of discrete finite element spaces for velocity and pressure in order to acquire a stable finite element scheme. Furthermore, it is common strategy in multiphase flow applications that the emerging immersed interface is not resolved by the Eulerian grid. Instead, the forces on the immersed interface get included into the discrete spaces.

In a previous work on a stable discretisation for particle-fluid flows, see [2], [3], we derived suitable finite element spaces for the description of the interface forces between fluid and particles. One essential ingredient is the comparison to an according finite volume scheme [1]. Since FVM comprise discretised surface integrals they turned out to be convenient to capture the interaction forces on the immersed interface.
The insights and ideas gained from that simplified model system was extended to the general case of fluid-fluid two phase flows. Within that talk we propose a finite element space which offers a new approach for the inclusion of forces arising on immersed interfaces. We will derive the distinguished properties of the defined shape functions and emphasize their positive impact on the numerical properties of the discrete scheme. As a proof of concept first numerical results will be presented.
Beside the application to multiphase flow the proposed spaces give rise to a new approach for the construction of inf-sup stable finite element spaces for velocity and pressure: pressure-like forces naturally arise due to the new degrees of freedom. The end of the talk gives a short outlook to that promising direction.

References:

[1] Bank, R. E. and Rose, D. J.: Some Error Estimates for the Box Method. *SIAM Journal on Numerical Analysis* 22:777–787 (1987).

[2] Hoellbacher, S.: Test space modeling for interface problems: A stable FV and FE scheme for the DNS of particulate flow. Part I: Rotational test spaces. (*submitted*)

[3] Hoellbacher, S.: Test space modeling for interface problems: Projected Discrete Delta Functions with Application to the DNS of particulate flow. Part II: Flat-Top test spaces. (*submitted*)

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