Robust Multigrid Methods for Isogeometric Discretizations of Multipatch Domains

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Isogeometric Analysis (IgA) is a novel approach for the discretization of partial differential equations, which is based on B-spline or NURBS ansatz functions and a representation of the computational domain by a global geometry function. More complicated domains cannot be represented by just one such geometry function. Instead, the whole domain is decomposed into patches, where each of those is represented by its own geometry function.

In IgA, we typically encounter as discretization parameters the mesh size and the spline degree. If linear solvers from standard finite elements are transferred to IgA in a naive way, typically their behaviour in the mesh size is as good as in the finite element case, but the performance deteriorates if the spline degree is increased. The same holds for multigrid solvers, where robustness in the grid size is not an issue, but standard smoothers (like Gauss Seidel) suffer from the exponential growth of the condition number of the linear system in the spline degree.

For the single patch case, the author and his coworkers have proposed a multigrid solver being robust both the grid size and the spline space. The solver exploits the tensor-product structure of the problem and robust approximation error and inverse estimates. In the talk we will see how domain decomposition approaches can be used to extend that smoother to the multipatch case. This yields methods showing robust convergence behaviour in the grid size and the spline degree. We will discuss how to develop a convergence theory that yields explicit bounds in terms of the grid size and the spline degree.

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