Pressure Robust Discretizations for Incompressible Flows

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Classical inf-sup stable mixed finite elements for the incompressible (Navier–)Stokes equations are not pressure-robust, i.e., their velocity errors depend on the continuous pressure. However, a modification only in the right hand side of a Stokes discretization is able to reestablish pressure-robustness, as shown recently for several inf-sup stable Stokes elements. For the modification of the right hand side a velocity reconstruction operator is constructed that maps discretely divergence-free test functions to exactly divergence-free ones. This can be done with a simple element wise BDM interpolator in the case of discontinuous pressure approximations. Recently, this concept was extended to low and high order Taylor–Hood and mini elements, which have continuous discrete pressures and also for a new discretization with a relaxed $H(\mathrm{div})$-conformity. We present the basic concept and show different numerical examples to confirm that the new pressure-robust elements converge with optimal order and outperform significantly the classical versions of those elements.

References:

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