Numerical Approximations of a Family of Nonlocal Operators on Bounded Domains

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The use of mathematical models involving fractional order derivatives to describe transport phenomena that deviate from the classical Markovian and Gaussian paradigm has been proposed in many different settings in the last few decades. A prototypical operator used to account for spatial nonlocality is the fractional Laplacian, which is naturally defined on unbounded domains. However, in many practical applications, the observed phenomena is confined to a particular bounded region in space. Hence, suitable modifications have to be made in the definition of the nonlocal operator in order to obtain a well-posed mathematical model, suitably accounting for the boundary conditions imposed by the physics of the observed problem. Motivated by the use of the spectral fractional Laplacian to describe spatial non locality on bounded domains, we study a discretisation method for this nonlocal operator based on a combination of the finite-element strategy and a quadrature formula for a singular one-dimensional integral. The proposed approach can be naturally applied to the case of more general nonlocal operators defined as fractional powers of their corresponding standard counterparts via their spectral expansion. A connection is made to fractional powers of non-negative definite matrices coming from the discretisation of the classical heat equation with suitable boundary conditions and some results for the considered approaches are shown.

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