A New Analysis of a Numerical Method for the Time-fractional Fokker-Planck Equation with General Forcing

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Two new convergence analyses are given for the finite element spatial discretization and piecewise-constant time discretization scheme that is used in [K. Le et al., *SIAM J. Numer. Anal.,* (54) 2016, pp.1763–1784] to solve the time-fractional Fokker-Planck equation on a domain $\Omega \times [0,T]$ with general forcing, i.e., where the forcing term is a function of both space and time. First, when the method is discretised only in space, stability and convergence are proved in a fractional norm that is stronger than the $L^2(\Omega)$ norm used in the above paper. Furthermore, unlike the bounds proved in Le et al., the constant multipliers in our analysis do not blow up as the order of the fractional derivative α approaches the classical value of 1. Second, when the method is discretised only in time, we present a new $L^2(\Omega)$ convergence proof that avoids a flaw in the proof of Theorem 4.4 of the Le et al. paper.

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