Goal-oriented Error Control for Stabilized Finite Element Methods

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The numerical approximation of nonstationary convection-diffusion-reaction problems

$$\partial_t u + \boldsymbol{b} \cdot \nabla u - \nabla \cdot (\varepsilon \nabla u) + \alpha u = f \tag{1}$$

with small diffusion $0 < \varepsilon \ll 1$ remains to be a challenging task. Eq. (1) is considered as a prototype model for more sophisticated equations of practical interest. For its numerical solution stabilized methods like the SUPG approach are used that aim to introduce a correct amount of artificial diffusion in regions with sharp inner or boundary layers or complicated structures where important physical or chemical phenomena take place.

Even though it seems to be natural to combine stabilized finite element methods with adaptive error control mechanisms to further enhance the approximation quality, this combination has been studied rarely so far in the literature. Existing a posteriori error analyses are either typically based on error norms that are non natural for the stabilized scheme or they are not robust with respect to the small diffusion parameter, i.e., that they involve constants that increase for vanishing diffusion. In this contribution we combine stabilized finite element methods with an a posteriori error control mechanism based on a dual weighted residual approach. The dual weighted error estimator assesses the discretization error with respect to a given goal quantity of physical interest. In contrast to former works on goal-oriented error control for transport problems, we solve the stabilized dual problem by a higher order approach and do not use a computationally less expensive higher order interpolation technique to determine the approximate dual solution. This is done in order to improve the approximation quality of the dual solution in the sensitive regions, i.e., in layers and regions with steep gradients. Thereby we aim to get an error representation for the goal quantity to the best feasible extent rather than an a posteriori error estimation.

The derivation of our goal-oriented error control for SUPG stabilized approximations of Eq. (1) is presented. Moreover, its numerical performance properties are studied and illustrated for benchmark problems of convection-dominated transport.

References:

[1] M. Bause, M. Bruchhäuser, K. Schwegler, *A goal-oriented a posteriori error control for unsteady convection-dominated problems*, **to appear**, 2017.

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