# Local Projection Stabilization for a Convection-Diffusion Equation on a Surface

Lutz Tobiska[1] Kristin Simon[2]

We consider the convection-diffusion equation

$$-\varepsilon\Delta_\Gamma u + \mathbf{b}\cdot\nabla_\Gamma u + cu = f \quad \text{on } \Gamma$$

posed on a hypersurface $\Gamma$. Here, $\nabla_\Gamma$ and $\Delta_\Gamma$ denote the surface gradient and the Laplace-Beltrami operator, respectively. The assumption

$$c(x) - \frac{1}{2}\left(\nabla_\Gamma\cdot\mathbf{b}\right)(x) \geq \sigma_0 > 0 \quad \text{for all } x\in\Gamma$$

guarantees the unique solvability of the associated weak formulation of the problem. As known for this type of equations when posed in a domain $\Omega\subset\mathbb{R}^d$, $d=2,3$, with boundary conditions, boundary and interior layers may occur and standard finite element methods tend to be unstable unless the mesh is sufficiently fine. Many approaches have been developed and studied to overcome these instabilities. In case of a transport equation on a closed surface $\Gamma$ much less is known. We propose a one-level local projection type stabilization and give an a priori error analysis in a mesh-dependent norm with error constants independent of $\varepsilon$.

[1] Institute for Analysis and Computational Mathematics, Otto von Guericke Magdeburg, Magdeburg, Germany, tobiska@ovgu.de

[2] Institute for Analysis and Computational Mathematics, Otto von Guericke Magdeburg, Magdeburg, Germany, kristin.simon@ovgu.de