Higher Order FEM for the Obstacle Problem of the p-Laplacian

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We consider two higher order finite element discretizations of an obstacle problem with the p-Laplacian differential operator for $p \in (1, \infty)$. The first approach is a non-linear variational inequality in the primal variable u only. The second formulation is a primal-dual mixed formulation where the dual variable represents the signed residual of the variational inequality from the first approach. These two formulations are equivalent and, under mild assumptions on the obstacle, even on the discrete level when using biorthogonal basis functions. We prove a priori error estimates as well as a general a posteriori error estimate which is valid for both formulations. We present numerical results on the improved convergence rates of adaptive schemes (mesh size adaptivity with and without polynomial degree adaptation) for the singular case $p = 1.5$ and the degenerated case $p = 3$. We also present numerical results on the mesh independency and on the polynomial degree scaling of the discrete inf-sup constant when using biorthogonal basis functions for the dual variable defined on the same mesh with the same polynomial degree distribution.

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