Local FEMs for the Fractional Laplacian

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We discuss several Finite Element Methods (FEMs) applied to the Caffarelli-Silvestre extension that localizes the fractional powers of symmetric, coercive,linear elliptic operators in bounded domains with Dirichlet boundary conditions. We consider open, bounded, polygonal not necessarily convex domains $\Omega \subset \mathbb{R}^2$. First, we discretize with continuous, piecewise linear, Lagrangian FEM (P_1-FEM) with mesh refinement near corners, and prove that the full convergence rate can be attained. Second, we also prove that tensorization of a P_1-FEM in Ω with a suitable hp-FEM in the extended variable achieves log-linear complexity with respect to the number of degrees of freedom in the domain Ω. Third, we propose a sparse tensor product FEM based on a multilevel P_1-FEM in Ω and on a P_1 FEM on radical–geometric meshes in the extended variable; this approach also achieves log-linear complexity with respect to N_Ω. Fourth, under stronger (analyticity) assumptions on the data (including the geometry Ω), we establish exponential rates of convergence of hp-FEM for spectral, fractional diffusion operators by discretizing with high order elements.

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