A Mixed Finite Element Approximation for the Compressible Euler Equations

Herbert Egger[1]

We consider the flow of compressible fluids through pipes and pipe networks. As a starting point, a thermodynamically consistent variational characterization of solutions to the one dimensional Euler equations is presented which very directly encodes the conservation of mass, energy, and entropy. This variational principle is suitable for a conforming Galerkin approximation in space which automatically inherits the basic physical conservation laws. A mixed finite element method is briefly discussed as a particular choice. We also investigate the discretization in time by a problem adapted implicit time stepping scheme for which we prove exact conservation of mass and a slight dissipation of energy and negative entropy. These deviations from the strict conservation laws are due to numerical dissipation of the implicit time discretization. The resulting fully discrete method can be extended naturally to more general flow models and also to pipe networks and is therefore well suited for the simulation of gas transport in pipelines.

[1] TU Darmstadt, Mathematics, Darmstadt, egger@mathematik.tu-darmstadt.de