A Black-Box Algorithm for Fast Matrix Assembly in Isogeometric Analysis

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A fast algorithm for assembling stiffness matrices for Isogeometric Analysis with tensor product spline spaces is presented. The procedure exploits the facts that (a) such matrices have block-banded structure, and (b) they often have low Kronecker rank. Combined, these two properties allow us to reorder the nonzero entries of the stiffness matrix into a relatively small, dense matrix or tensor of low rank. A suitable black-box low-rank approximation algorithm is then applied to this matrix or tensor. This allows us to approximate the nonzero entries of the stiffness matrix while explicitly computing only relatively few of them, leading to a fast assembly procedure.

The algorithm does not require any further knowledge of the used spline spaces, the geometry transform, or the partial differential equation, and thus is black-box in nature. Existing assembling routines can be reused with minor modifications. A reference implementation is provided which can be integrated into existing code.

In several numerical examples, we demonstrate significant speedups over a standard Gauss quadrature assembler for several geometries in two and three dimensions.

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