Optimal Error Analysis of a Direct Discontinuous Galerkin Method for Time-fractional Reaction-Diffusion Equation

Chaobao Huang[1] Martin Stynes Na An

A time-dependent reaction-diffusion initial-boundary value problem with a Caputo time derivative of order $\alpha \in (0,1)$ is considered. Its solution has a weak singularity at the initial time $t = 0$. Bounds on certain derivatives of the solution are obtained. A fully discrete Direct Discontinuous Galerkin (DDG) method that is designed to deal with this initial singularity is presented and analysed. In this method the well-known L1 scheme on a graded mesh is used for the time discretisation, while a DDG method on a uniform mesh is used in the spatial direction. Then L^2-norm stability and consistency estimates are derived for the method; during this analysis, a new projection is developed to handle the Dirichlet boundary conditions—it is shown to be well defined (which is a non-trivial result) and bounds on the projection error are derived in various norms. Using this information, an optimal L^2-norm error estimate is obtained. Numerical experiments are presented that confirm the sharpness of the error analysis.

[1] Beijing Computational Science Research Center, Applied and Computational Mathematics Division, Beijing, China, huangcb@csrc.ac.cn