Adaptive Stochastic Galerkin Methods for Parametric PDEs with Spatial Singularities

Alex Bespalov[1] Leonardo Rocchi[2]

We consider a class of elliptic PDEs where the underlying differential operator has affine dependence on a large, possibly infinite, number of random parameters. Stochastic Galerkin Finite Element Methods (sGFEM) have emerged over the last decade as an efficient alternative to sampling methods for numerical solution of such problems. The sGFEM approximations are sought in tensor product spaces $X \otimes P$, where X is a conventional finite element space associated with a physical domain and P is a finite-dimensional space of multivariate polynomials in the parameters.

If a large number of random parameters is used to represent the input data and highly refined spatial grids are used for finite element approximations on the physical domain, then computing the sGFEM solution becomes prohibitively expensive, due to huge dimension of the space $X \otimes P$. One way to avoid this is to use an adaptive approach, in which spatial (X-) and stochastic (P-) components of approximations are judiciously chosen and incrementally refined in the course of numerical computation. Adaptive refinement of spatial approximations is particularly important when solutions exhibit singularities (e.g., due to geometry of the physical domain).

In this talk, we present an adaptive algorithm implementing stochastic Galerkin FEM. Building upon theoretical results in [1, 2] we employ a hierarchical a posteriori error estimation strategy to control the energy error and use the estimates of error reduction to steer adaptive refinement of spatial and stochastic components of Galerkin approximations. The results of numerical experiments for representative model problems with parametric coefficients and spatially singular solutions will be discussed. These results demonstrate the effectiveness of our adaptive refinement strategy.

References:

[1] A. Bespalov, C. E. Powell, and D. Silvester, Energy norm a posteriori error estimation for parametric operator equations, SIAM J. Sci. Comput., 36 (2014), pp. A339–A363.

[2] A. Bespalov and D. Silvester, Efficient adaptive stochastic Galerkin methods for parametric operator equations, SIAM J. Sci. Comput., 38 (2016), pp. A2118–A2140.

[1] School of Mathematics, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom, a.bespalov@bham.ac.uk

[2] School of Mathematics, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom, LXR507@student.bham.ac.uk