Discontinuous Galerkin Time Stepping for Parabolic Evolution Problems

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The discontinuous Galerkin (dG) time marching scheme has become a popular method for the discretization of parabolic problems. In semi-discrete form it is based on a temporal FE-type formulation (of arbitrary order) that naturally inherits the structure of the underlying evolution problem. In particular, this framework enables the application of classical FE concepts including, for instance, the use of local refinements (with respect to both the time step lengths as well as the approximation orders), or the development of computable a posteriori error estimates.

In this talk we will present a new technique that provides a discrete calculus for the dG time stepping scheme. As a consequence, we will be able to prove discrete Peano-type existence results for the dG method in the context of very general (linear and nonlinear) initial value problems. Furthermore, in combination with an elliptic reconstruction technique, we will show how fully computable a posteriori $L^2(H^1)$- and $C^0(L^2)$-type error bounds for a fully-discrete method consisting of the dG time stepping scheme and a standard conforming FEM in space for linear parabolic PDE can be derived.

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