Analysis of Galerkin and SDFEM on Layer-Adapted Meshes for Turning Point Problems Exhibiting an Interior Layer

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We consider linear second order singularly perturbed boundary value problems on an interval with an interior turning point x_0. The turning point – zero of the convection factor – is presumed to be simple and attractive. In this case it is well known that the problem solution exhibits in general an interior layer of "cusp"-type at x_0. Therefore, standard finite element methods on uniform meshes do not converge properly.

In this talk, we present layer-adapted meshes that enable to prove optimal convergence rates in the energy norm for finite elements of order $k \geq 1$ uniform with respect to the perturbation parameter ε. However, in numerical experiments still non-physical oscillations in the error can be observed. In order to increase stability and to damp such behaviour we additionally use and analyse the streamline diffusion finite element method (SDFEM) on these meshes.

Finally, numerical experiments are given to illustrate and confirm the theoretical findings.

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