Energy-Correction Method for Parabolic Problems

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Let $\Omega \subset \mathbb{R}^2$ be a bounded polygonal domain with a re-entrant corner, i.e. corner with an angle $\Theta > \pi$. In this talk we consider a model linear heat equation and investigate the behaviour of its piecewise linear finite element approximation in space. Its spatial convergence order is lower due to reduced regularity in the presence of the re-entrant corner. Recently, an effective method of recovering full second-order convergence for elliptic equations on domains with re-entrant corners, when measured in weighted L_2 and H^1 norms, known as energy-correction, has been proposed. This method is based on a modification of a fixed number of entries in the system's stiffness matrix. We show how energy-cor6rection method can be successfully applied to regain optimal convergence in weighted norms for parabolic problems. Moreover, we propose a post-processing strategy yielding optimal convergence order in standard Sobolev norms.

Standard discretisation approach involving graded meshes results in a very restrictive form of a CFL condition, making the use of explicit time stepping practically impossible. On the other hand, energy-corrected finite element can be used on uniform meshes allowing for application of explicit time stepping scheme with relatively large time steps. This, combined with mass-lumping strategy, leads to a very efficient discretisation of parabolic problems, where at each time step only one vector multiplication with a scaled stiffness matrix needs to be performed.

All theoretical results are confirmed by numerical test.

References:

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