Extended Finite Element Methods Dealing with Singularities on Meshes of Combined Dimensions

Pavel Exner[1] Jan Březina[2]

Our research is aimed at modelling of groundwater flow with finite element method on meshes of combined dimensions. General problems of these models (also in many other applications) is disability to capture small-scale phenomena in large domains and creation of suitable conforming meshes in real world situations. The extended finite element method (XFEM) provides a way to overcome these problems. However, its usage in models with combined dimensions is still developing and has many open questions. The main goal is to propose and implement a proper XFEM enrichment in a model of groundwater flow, governed by Darcy's law, to enable coupling between dimensions on non-conforming meshes. In particular, we are interested in 0D and 1D singularities in 2D and 3D, respectively.

We study different enrichment methods for pressure in a steady well-aquifer problem. The finite element space is enriched with an analytical solution to a local Laplace problem with a point source. We develop accurate quadrature rules for integration over enriched elements to obtain the optimal order of convergence. We also investigate a reasonable choice of the enriched area.

Further, we aim our effort at usage of the XFEM in the software Flow123d, a simulator of underground water flow and solute transport on meshes of combined dimension. Since the knowledge of an accurate velocity field is of a great importance in the transport equation, the mixed-hybrid form of the problem is used with the lowest order Raviart-Thomas finite elements. We are interested in finding a suitable enrichment both for pressure and velocity, however, the satisfaction of the inf-sup condition in this case is in question.

References:

[1] P. Exner, J. Březina, Partition of unity methods for approximation of point water sources in porous media, Applied Mathematics and Computation 273C (2016) 21-32. doi:10.1016/j.amc.2015.09.048.
[2] J. Šístek, J. Březina, B. Sousedík, BDDC for mixed-hybrid formulation of flow in porous media with combined mesh dimensions, Numerical Linear Algebra with Applications (2015). doi:10.1002/nla.1991.
[3] J. R. Craig, R. Gracie, Using the extended finite element method for simulation of transient well leakage in multilayer aquifers, Advances in Water Resources 34 (9) (2011) 1207-1214. doi:10.1016/j.advwatres.2011.04.004.
[4] T.-P. Fries, A corrected XFEM approximation without problems in blending elements, International Journal for Numerical Methods in Engineering 75 (5) (2008) 503-532. doi:10.1002/nme.2259.
[5] I. Babuška, U. Banerjee, Stable generalized finite element method (SGFEM), Computer Methods in Applied Mechanics and Engineering (2012) 91-111. doi:10.1016/j.cma.2011.09.012.

[1] Technical University of Liberec, Studentska 1402/2, 46117 Liberec, Czech Republic, Technische Universität München, Lehrstuhl für Numerische Mathematik, Garching bei München, Germany,
pavelexner@gmail.com

[2] Technical University of Liberec, Studentská 1402/2, 46117 Liberec, Czech Republic,
jan.brezina@tul.cz