Weighted Finite Element Error Estimates on Boundary Concentrated Meshes with Application to Dirichlet Control

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In this talk we present new error estimates in special $L^2(\Omega)$-norms containing the distance towards the boundary as weight function. These error estimates are of interest when deriving a priori estimates for the approximate normal derivative of elliptic equations, and, the application that we want to focus on, for the numerical approximation of Dirichlet control problems where the control variable corresponds to the normal derivative of some adjoint variable. Our aim is to improve the accuracy of the numerical approximation by refining the computational meshes towards the boundary of the underlying domain.

We restrict our considerations to the Poisson equation with Dirichlet boundary conditions in convex polygonal domains. As the vertices in the domain cause also singularities we have to deal with regularity results in weighted Sobolev spaces containing the distance to the vertices and the boundary as weight functions. To prove error estimates we extend a technique that is used to prove local maximum norm estimates, more precisely, we apply a combination of dyadic decompositions with respect to the element boundary and the vertices of the computational domain to carve out both types of singularities. As a main result we show that the convergence rate of almost two (up to logarithmic factors) can be achieved also in weighted $L^2(\Omega)$-norms provided that the vertex singularities are mild enough. As a by-product we will observe that singularities corresponding to vertices having interior angle less than 120° do not lower the convergence rates. If there are vertices having a larger opening angle we prove lower convergence rates that depend solely on those angles. The predicted behavior is confirmed in our numerical experiments.

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