Local Projection Stabilization for Surface Transport Problems

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We consider a transport equation on a given surface. Similarly to the bulk setting finite element methods for transport equations on surfaces can lead to unphysical oscillations at internal and boundary layers and hence stabilization is needed. Olshanskii et al. presented a Streamline Upwind Petrov-Galerkin approach for linear finite elements on unfitted meshes in 2014.

We pursue another stabilization approach, called Local Projection Stabilization, on fitted approximations of the surface. Local Projection Stabilization was introduced by Becker and Braack in 2001 and is well studied for transport equations in the bulk, compare e.g. Ganesan and Tobiska (2010). By transferring this method to surface equations new challenges have to be handled. Amongst others geometric errors occur due to the non smooth approximation of the curved surface, given data and operators have to be defined on the discrete surface and partial integration on the discrete surface leads to additional integrals over element boundaries.

A uniquely solvable formulation of the stabilized problem is introduced. Estimations of the geometric and consistency error are shown and the gained order of convergence is compared to the standard Galerkin method. Numerical experiments show the stabilizing properties of the proposed method.

References:

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