Variational Methods for Radiative Transfer

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Radiative transfer describes the propagation, absorption, and scattering of electromagnetic radiation traversing a turbid background medium. A typical example is light propagation through cloudy atmospheres or in biological tissue.

The basic mathematical model consists of an integro-partial differential equation in three angular, three spatial and one time variable. The radiative transfer equation governs the evolution of the spectral radiance. Similar mathematical models also arise in neutron transport or linearized particle dynamics.

In this talk, we present a variational framework for radiative transfer that allows a rigorous analysis of the problem on the analytic level and a systematic discretization by Galerkin methods in angle, space, and time. Existence and uniqueness of solutions are proven on the continuous and the discrete level in the framework of mixed variational problems. We briefly discuss asymptotic regimes and present computational results obtained with a particular discretization based on a truncated spherical harmonics expansion in angle, mixed finite element approximations in space, and a discontinuous Galerkin method in time.

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