Numerical Solution of Boundary Value Problems on Domains with a Thin Layer of Random Thickness

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The present talk is dedicated to the numerical solution of boundary value problems on domains with a thin layer of different conductivity and of random thickness. By changing the boundary condition, the boundary value problem given on the random domain can be transformed into a boundary value problem on a fixed domain. The randomness is then contained in the coefficients of the new boundary condition. The solution of this new boundary value problem approximates the original solution with leading order in the scale parameter ε of the layer's thickness. With the help of the Karhunen-Loève expansion, we transform this random boundary value problem into a deterministic, parametric one with a possibly high-dimensional parameter $\mathbf{y}$. Based on the decay of the random fluctuations of the layer's thickness, we prove rates of decay of the derivatives of the random solution with respect to this parameter $\mathbf{y}$ which are robust in the scale parameter ε. Numerical results validate our theoretical findings.

References:

[1] M. Dambrine, I. Greff, H. Harbrecht, and B. Puig. Numerical solution of the Poisson equation with a thin layer of random thickness. SIAM J. Numer. Anal., 54(2):921–941, 2016.
[2] M. Dambrine, I. Greff, H. Harbrecht, and B. Puig. Numerical solution of the homogeneous Neumann boundary value problem on domains with a thin layer of random thickness. Preprint 2016-06, Mathematisches Institut, Universität Basel, Switzerland, 2016.

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