Semi-Robust Error Estimates of Galerkin-FEM with Exactly Divergence-Free Finite Elements for Incompressible Flows

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We consider the Galerkin-FEM for the time-dependent incompressible Navier-Stokes equations: *Find* $(\mathbf{u}, p) \in V \times Q \equiv [W_0^{1,2}(\Omega)]^d \times L_0^2(\Omega)$ *s.t.*

$$\begin{aligned} \partial_t \mathbf{u} - \nu \Delta \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p &= \mathbf{f}, \quad (1) \\ \nabla \cdot \mathbf{u} &= 0. \quad (2) \end{aligned}$$

As starting point, we apply Scott-Vogelius element pairs $[\mathbf{P}^{k+1}]^d \times \mathbf{P}^{-k}$ with $k \geq d$ on barycentrically refined simplicial meshes for the discrete velocity and pressure spaces $V_h \times Q_h$ which are known to safisfy the constraint (2) even pointwise. In case of $\mathbf{u} \in L^\infty(0, T; W^{1,\infty}(\Omega)^d)$, we extend the semi-robust semidiscrete error estimates of [1] to the given case. In particular, we point out the gain of pressure-robust schemes in the sense of [2].

Then we discuss an extension to $H(div)$-conforming dG-methods in $V_h \times Q_h$ with $\nabla \cdot V_h \subseteq Q_h$ together with (symmetric) interior penalty and upwind stabilization.

Finally, we present and discuss numerical simulations for basic benchmark problems like Euler vortex dynamics and Blasius boundary layer flow.

References:

[1] H. Dallmann, D. Arndt: Stabilized finite element methods for the Oberbeck/Boussinesq model. J. Sc. Comput. 2016, DOI 10.1007/s 10915-016-0191-z

[2] V. John, A. Linke, C. Merdon, M. Neilan, L. Rebholz: On the divergence constraint in mixed finite element methods for incompressible flows. WIAS-Preprint 2177, 2015.

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