Error Estimates for the Numerical Approximation of Sparse Parabolic Control Problems

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In this talk we consider the following optimal control problem

$$\text{(P)} \qquad \min_{u \in L^\infty(Q)} J(u) = F(u) + \mu j(u),$$

where

$$F(u) = \frac{1}{2}\int_Q (y_u(x,t) - y_d(x,t))^2\,dx\,dt + \frac{\nu}{2}\int_Q u^2(x,t)\,dx\,dt,$$

$$j(u) = \int_\Omega \left(\int_0^T u^2(x,t)\,dt\right)^{1/2} dx,$$

and y_u is the state associated to the control u, solution of

$$\begin{cases} \partial_t y - \Delta y + a(x,t,y) = u \ \text{ in } \ Q = \Omega \times (0,T), \\ y = 0 \ \text{ on } \ \Sigma = \Gamma \times (0,T), \\ y(0) = y_0 \ \text{ in } \ \Omega. \end{cases}$$

We prove the existence of at least one solution to problem (P), and establish the first and second order optimality conditions. From the first order optimality conditions we deduce the spatial sparsity of the optimal controls as well as some regularity properties. Finally, we define a finite-element based approximation of the control problem and, assuming a second order sufficient optimality condition, we prove L^2 error estimates for the difference between the locally optimal controls and their discrete approximations.

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