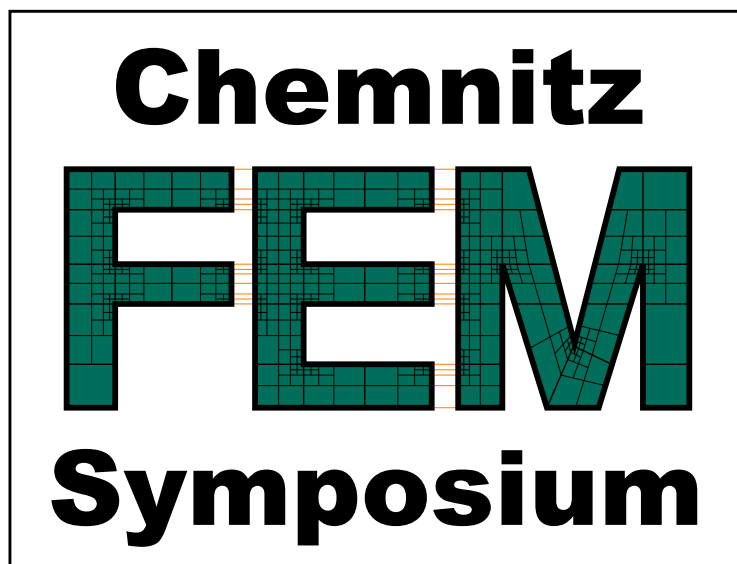




TECHNISCHE UNIVERSITÄT
CHEMNITZ

Fakultät für Mathematik

Chemnitz FEM-Symposium 2016



Programme

Collection of abstracts

List of participants

Chemnitz, September 26 - 28, 2016

Scientific Topics:

The symposium is devoted to all aspects of finite elements and wavelet methods in partial differential equations.

The topics include (but are not limited to):

- adaptive methods,
- parallel implementation,
- high order methods.

This year we particularly encourage talks on:

- optimal control and FE,
- DG and space-time methods,
- Maxwell's equations and piezoelectric problems.

Invited Speakers:

Eduardo Casas (University of Cantabria)

Herbert Egger (TU Darmstadt)

Manfred Kaltenbacher (Vienna University of Technology)

Thomas Wihler (University of Bern)

Conference Venue:

Hotel an der Oper
Straße der Nationen 56
09111 Chemnitz, Germany
<http://www.hoteloper-chemnitz.de>

Scientific Committee:

Th. Apel (München), S. Beuchler (Bonn), O. Ernst (Chemnitz), G. Haase (Graz),
H. Harbrecht (Basel), R. Herzog (Chemnitz), M. Jung (Dresden), U. Langer (Linz),
A. Meyer (Chemnitz), A. Rösch (Duisburg-Essen), O. Steinbach (Graz)

Organising Committee:

B. Sprungk, F. Ospald, M. Weise, R. Springer, M. Pester, A.-K. Glanzberg, K. Seidel

<https://www.tu-chemnitz.de/mathematik/fem-symposium/>



Programme

Programme for Monday, September 26, 2016

9:00	Opening	Arnd Meyer
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9:55	Markus Bause 9 Space-Time Finite Element Approximation of Fluid-Structure Interaction in Porous Media.	
10:20	Friedhelm Schieweck / Gunar Matthies 10 The Role of Post-Processing in Variational Time Discretizations.	
10:45	<i>Tea and coffee break</i>	
	Convection Dominated Problems	<i>Chair:</i> Thomas Wihler
11:15	Gert Lube 11 Semi-Robust Error Estimates of Galerkin-FEM with Exactly Divergence-Free Finite Elements for Incompressible Flows .	
11:40	Simon Becher 12 Analysis of Galerkin and SDFEM on Layer-Adapted Meshes for Turning Point Problems Exhibiting an Interior Layer.	
12:05	Piotr Skrzypacz 13 A Way to Improve the Solution of Local Projection Stabilization.	
12:30	Kristin Simon 14 Local Projection Stabilization for Surface Transport Problems.	
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High-Dimensional Problems

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 Variational Methods for Radiative Transfer.
- 15:20 **Jürgen Dölz** 16
 H-Matrix Based Second Moment Analysis for Rough Random Fields.
- 15:45 **Helmut Harbrecht** 17
 Numerical Solution of Boundary Value Problems on Domains with a Thin Layer of Random Thickness.

16:10 *Tea and coffee break*

Computational Mechanics

Chair: Herbert Egger

- 16:40 **Fleurianne Bertrand** 18
 Least-Squares Methods for Elasticity with Weakly Imposed Symmetry.
- 17:05 **Hansjörg Schmidt** 19
 Symmetric Mixed Formulation for Reinforced Elastomers.
- 17:30 **Rolf Springer** 20
 Efficient Simulation of Short Fibre Reinforced Composites.
- 17:55 **Michael Weise** 21
 Locking in Variations of the Mindlin-Reissner Plate Model.

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Programme for Tuesday, September 27, 2016

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9:50	Ioannis Touloupoulos 23 Discontinuous Galerkin Isogeometric Analysis on Decompositions with Gaps and Overlaps.	
10:15	Gundolf Haase 24 Fast Realization of Eikonal Equation Solvers on Tablet Devices.	
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10:40	<i>Tea and coffee break</i>	
Singularities		<i>Chair: Manfred Kaltenbacher</i>
11:10	Viktor Rukavishnikov 25 Weighted Finite Element Method for the Elasticity Problem with Singularity.	
11:35	Piotr Swierczynski 26 Energy-Correction Method for Parabolic Problems.	
12:00	Pavel Exner 27 Extended Finite Element Methods Dealing with Singularities on Meshes of Combined Dimensions.	
12:25	Marcel Moldenhauer 28 Stress Reconstruction for the Nonconforming P2 Finite Element Method and A Posteriori Error Estimation.	
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14:00	<i>Conference Photo / Excursion</i>	
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Optimal Control		<i>Chair: Roland Herzog</i>
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9:50	Arnd Rösch 30 Piecewise Linear Control Functions for Semilinear Optimal Control Problems with Directional Sparsity.	
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Optimal Control		<i>Chair: Arnd Rösch</i>
11:10	Sven Beuchler 32 Fast Solvers for hp-FEM Discretized PDE-Constrained Optimization Problems.	
11:35	Gerd Wachsmuth 33 Discretization of Control Constrained Optimal Control Problems with Mass Lumping.	
12:00	Thomas Apel 34 Discretization Error Estimates for Dirichlet Control Problems in Polygonal Domains.	
12:25	Max Winkler 35 Weighted Finite Element Error Estimates on Boundary Concentrated Meshes with Application to Dirichlet Control.	
12:50	Closing	
13:00	<i>Lunch</i>	

Collection of abstracts

Discontinuous Galerkin Time Stepping for Parabolic Evolution Problems

Thomas P. Wihler¹

The discontinuous Galerkin (dG) time marching scheme has become a popular method for the discretization of parabolic problems. In semi-discrete form it is based on a temporal FE-type formulation (of arbitrary order) that naturally inherits the structure of the underlying evolution problem. In particular, this framework enables the application of classical FE concepts including, for instance, the use of local refinements (with respect to both the time step lengths as well as the approximation orders), or the development of computable a posteriori error estimates.

In this talk we will present a new technique that provides a discrete calculus for the dG time stepping scheme. As a consequence, we will be able to prove discrete Peano-type existence results for the dG method in the context of very general (linear and nonlinear) initial value problems. Furthermore, in combination with an elliptic reconstruction technique, we will show how fully computable a posteriori $L^2(H^1)$ - and $C^0(L^2)$ -type error bounds for a fully-discrete method consisting of the dG time stepping scheme and a standard conforming FEM in space for linear parabolic PDE can be derived.

¹ University of Bern, Mathematics Institute, Bern, Switzerland,
wihler@math.unibe.ch

Space-Time Finite Element Approximation of Fluid-Structure Interaction in Porous Media

Markus Bause¹ Uwe Köcher²

Space-time finite element methods are getting of increasing importance for the accurate numerical approximation of solutions to coupled systems of partial differential equations with complex behavior in space and time. The schemes offer appreciable advantages in the discretization of such problems if strong coupling mechanisms and memory terms are involved. Moreover, they allow the natural construction of higher order methods and the applicability of adaptive finite element techniques in space and time. We present and study three families of continuous, discontinuous and continuously differentiable Galerkin time discretization schemes. Along with discontinuous Galerkin discretizations of the spatial variables they are used firstly for the simulation of hyperbolic wave propagation and elastic deformations in multiscale material. As an extension, the potential of the schemes for solving the problem of flow in porous media undergoing a fluid-structure interaction and mechanical deformations of the solid material is studied afterwards. Results from the numerical analyses of the schemes, algorithmic aspects as well as numerical experiments are presented.

References:

- [1] M. Bause, U. Köcher, *Iterative coupling methods for poroelasticity*, ENUMATH 2015 Proceedings, Springer, 2016.
- [2] M. Bause, U. Köcher, *Variational time discretization for mixed FEM*, J. Comp. Appl. Math., **289** (2015), 208–224.
- [3] U. Köcher, M. Bause, *Variational space-time methods for the wave equation*, J. Sci. Comp., **61** (2014), 424–453.

¹ Helmut Schmidt University, Faculty of Mechanical Engineering, Hamburg, Germany,
bause@hsu-hh.de

² Helmut Schmidt University, Faculty of Mechanical Engineering ,
uwe.koecher@hsu-hh.de

The Role of Post-Processing in Variational Time Discretizations

Friedhelm Schieweck¹ Gunar Matthies²

We consider the discontinuous Galerkin (dG) method and the continuous Galerkin-Petrov (cGP) method as variational time discretizations for transient partial differential equations. For simplicity of presentation, we demonstrate the ideas for the heat equation and the time-dependent Stokes problem which are discretized in space by a finite element method. In case of the Stokes problem, we use an inf-sup stable pair of finite element spaces for approximating velocity and pressure.

Once the fully discrete solution with dG or cGP has been computed on a time interval, it can be lifted at low computational costs to a post-processed solution which is a time polynomial of one degree higher and provides a higher global smoothness with respect to time. We obtain for dG a global C^0 approximation in time and for cGP an approximation which is globally C^1 . We will present for the post-processed solution optimal error estimates in the $L^2(L^2)$ -norm which are of higher order in time. Numerical results confirming the theoretical predictions will be given.

¹ Institut für Analysis und Numerik, Otto-von-Guericke Universität Magdeburg, Postfach 4120, D-39016 Magdeburg, Germany,
schiewec@ovgu.de

² Technische Universität Dresden, Institut für Numerische Mathematik, 01062 Dresden,
gunar.matthies@tu-dresden.de

Semi-Robust Error Estimates of Galerkin-FEM with Exactly Divergence-Free Finite Elements for Incompressible Flows

Gert Lube¹ Philipp Schroeder²

We consider the Galerkin-FEM for the time-dependent incompressible Navier-Stokes equations: Find $(\mathbf{u}, p) \in V \times Q \equiv [W_0^{1,2}(\Omega)]^d \times L_0^2(\Omega)$ s.t.

$$\partial_t \mathbf{u} - \nu \Delta \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p = \mathbf{f}, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0. \quad (2)$$

As starting point, we apply Scott-Vogelius element pairs $[\mathbf{P}^{k+1}]^d \times \mathbf{P}^{-k}$ with $k \geq d$ on barycentrically refined simplicial meshes for the discrete velocity and pressure spaces $V_h \times Q_h$ which are known to satisfy the constraint (2) even pointwise. In case of $\mathbf{u} \in L^\infty(0, T; W^{1,\infty}(\Omega)^d)$, we extend the semi-robust semidiscrete error estimates of [1] to the given case. In particular, we point out the gain of pressure-robust schemes in the sense of [2].

Then we discuss an extension to $H(\text{div})$ -conforming dG-methods in $V_h \times Q_h$ with $\nabla \cdot V_h \subseteq Q_h$ together with (symmetric) interior penalty and upwind stabilization.

Finally, we present and discuss numerical simulations for basic benchmark problems like Euler vortex dynamics and Blasius boundary layer flow.

References:

- [1] H. Dallmann, D. Arndt: Stabilized finite element methods for the Oberbeck/Boussinesq model. J. Sc. Comput. 2016, DOI 10.1007/s 10915-016-0191-z
- [2] V. John, A. Linke, C. Merdon, M. Neilan, L. Rebholz: On the divergence constraint in mixed finite element methods for incompressible flows. WIAS-Preprint 2177, 2015.

¹ Georg-August-University Göttingen, Institute for Numerical and Applied Mathematics, Lotzestraße 16-18, D-37083 Göttingen, Germany,
lube@math.uni-goettingen.de

² Georg-August-University Göttingen, Institute for Numerical and Applied Mathematics, Lotzestraße 16-18, D-37083 Göttingen, Germany,
p.schroeder@math.uni-goettingen.de

Analysis of Galerkin and SDFEM on Layer-Adapted Meshes for Turning Point Problems Exhibiting an Interior Layer

Simon Becher¹

We consider linear second order singularly perturbed boundary value problems on an interval with an interior turning point x_0 . The turning point – zero of the convection factor – is presumed to be simple and attractive. In this case it is well known that the problem solution exhibits in general an interior layer of “cusp”-type at x_0 . Therefore, standard finite element methods on uniform meshes do not converge properly.

In this talk, we present layer-adapted meshes that enable to prove optimal convergence rates in the energy norm for finite elements of order $k \geq 1$ uniform with respect to the perturbation parameter ε . However, in numerical experiments still non-physical oscillations in the error can be observed. In order to increase stability and to damp such behaviour we additionally use and analyse the streamline diffusion finite element method (SDFEM) on these meshes.

Finally, numerical experiments are given to illustrate and confirm the theoretical findings.

¹ Technische Universität Dresden, Institut für Numerische Mathematik, Dresden, Germany,
Simon.Becher@tu-dresden.de

A Way to Improve the Solution of Local Projection Stabilization

Piotr Skrzypacz¹ Friedhelm Schieweck²

Composite finite elements and novel postprocessing based on the local L_2 projection are proposed in order to improve the solution of standard one-level Local Projection Stabilization (LPS) on quadrilateral meshes, cf. [1,2,3]. Theoretical results are justified by several tests for convection-dominated problems in two dimensions. Numerical results show that the discrete solution is oscillation-free and of optimal accuracy in the regions away from the boundary layer whereas the spurious oscillations are significantly reduced near the boundary layers when the postprocessing is applied.

References:

- [1] *Matthies Gunar, Skrzypacz Piotr, and Tobiska Lutz* A unified convergence analysis for local projection stabilisations applied to the Oseen problem. *M2AN Math. Model. Numer. Anal.*, 41(4):713-742, 2007.
- [2] *Matthies Gunar, Skrzypacz Piotr, and Tobiska Lutz* Stabilization of local projection type applied to convection-diffusion problems with mixed boundary conditions. *Electron. Trans. Numer. Anal.* 32 (2008), 90–105.
- [3] *Schieweck Friedhelm and Skrzypacz Piotr* A local projection stabilization method with shock capturing and diagonal mass matrix for solving non-stationary transport dominated problems. *Comput. Methods Appl. Math.* 12 (2012), no. 2, 221–240.

¹ School of Science and Technology, Nazarbayev University, Kazakhstan, Astana,
`piotr.skrzypacz@nu.edu.kz`

² Otto von Guericke University of Magdeburg, Department of Mathematics, Germany, Magdeburg,
`schiewec@ovgu.de`

Local Projection Stabilization for Surface Transport Problems

Kristin Simon¹ Lutz Tobiska²

We consider a transport equation on a given surface. Similarly to the bulk setting finite element methods for transport equations on surfaces can lead to unphysical oscillations at internal and boundary layers and hence stabilization is needed. Olshanskii et al. presented a Streamline Upwind Petrov-Galerkin approach for linear finite elements on unfitted meshes in 2014.

We pursue another stabilization approach, called Local Projection Stabilization, on fitted approximations of the surface. Local Projection Stabilization was introduced by Becker and Braack in 2001 and is well studied for transport equations in the bulk, compare e.g. Ganesan and Tobiska (2010). By transferring this method to surface equations new challenges have to be handled. Amongst others geometric errors occur due to the non smooth approximation of the curved surface, given data and operators have to be defined on the discrete surface and partial integration on the discrete surface leads to additional integrals over element boundaries.

A uniquely solvable formulation of the stabilized problem is introduced. Estimations of the geometric and consistency error are shown and the gained order of convergence is compared to the standard Galerkin method. Numerical experiments show the stabilizing properties of the proposed method.

References:

- [1] Becker, Roland, and Malte Braack. *A finite element pressure gradient stabilization for the Stokes equations based on local projections*. *Calcolo* 38.4 (2001): 173-199.
- [2] Ganesan, Sashikumar, and Lutz Tobiska. *Stabilization by local projection for convection-diffusion and incompressible flow problems*. *Journal of Scientific Computing* 43.3 (2010): 326-342.
- [3] Olshanskii, Maxim A., Arnold Reusken, and Xianmin Xu. *A stabilized finite element method for advection-diffusion equations on surfaces*. *IMA journal of numerical analysis* 34.2 (2014): 732-758.

¹ Otto-von-Guericke Universität Magdeburg, Institut für Analysis und Numerik, Magdeburg, Germany,
Kristin.Simon@ovgu.de

² Otto-von-Guericke Universität Magdeburg, Institut für Analysis und Numerik,
lutz.tobiska@ovgu.de

Variational Methods for Radiative Transfer

Herbert Egger¹

Radiative transfer describes the propagation, absorption, and scattering of electromagnetic radiation traversing a turbid background medium. A typical example is light propagation through cloudy atmospheres or in biological tissue.

The basic mathematical model consists of an integro-partial differential equation in three angular, three spatial and one time variable. The radiative transfer equation governs the evolution of the spectral radiance. Similar mathematical models also arise in neutron transport or linearized particle dynamics.

In this talk, we present a variational framework for radiative transfer that allows a rigorous analysis of the problem on the analytic level and a systematic discretization by Galerkin methods in angle, space, and time. Existence and uniqueness of solutions are proven on the continuous and the discrete level in the framework of mixed variational problems. We briefly discuss asymptotic regimes and present computational results obtained with a particular discretization based on a truncated spherical harmonics expansion in angle, mixed finite element approximations in space, and a discontinuous Galerkin method in time.

¹ Technische Universität Darmstadt, Fachbereich Mathematik, AG Numerik und Wissenschaftliches Rechnen,
Darmstadt, Germany,
egger@mathematik.tu-darmstadt.de

H-Matrix Based Second Moment Analysis for Rough Random Fields

Jürgen Dölz¹ Helmut Harbrecht² Michael Peters³ Christoph Schwab⁴

The efficient solution of operator equations with random right hand side is considered. The solution's two-point correlation can efficiently be computed by means of a sparse grid or a low-rank approximation if the two-point correlation of the right hand side is sufficiently smooth. Unfortunately, the problem becomes much more involved in case of rough data. However, the rough data and also the inverse operators can efficiently be represented or approximated by means of H-matrices. This enables us to solve the correspondent H-matrix equation in almost linear time by the use of the H-matrix arithmetic. Numerical experiments stemming from partial differential equations with random input data discretized by the finite element and the boundary element method are provided to validate and quantify the presented methods and algorithms.

References:

- [1] J. Dölz, H. Harbrecht, and M. Peters. H-matrix accelerated second moment analysis for potentials with rough correlation. *J. Sci. Comput.*, 65(1):387-410 (2015).
- [2] J. Dölz, H. Harbrecht, and C. Schwab. Covariance regularity and H-matrix approximation for rough random fields. Preprint 2014-11, Mathematisches Institut, Universität Basel, Switzerland, 2014.
- [3] J. Dölz, H. Harbrecht, and M. Peters. H-matrix based second moment analysis for rough random fields and finite element discretizations. Preprint 2015-35, Mathematisches Institut, Universität Basel, Switzerland, 2015.

¹ Universität Basel, Departement für Mathematik und Informatik, Spiegelgasse 1, 4053 Basel, juergen.doelz@unibas.ch

² Universität Basel, Departement für Mathematik und Informatik, Spiegelgasse 1, 4053 Basel, helmut.harbrecht@unibas.ch

³ Universität Basel, Departement für Mathematik und Informatik, Spiegelgasse 1, 4053 Basel, michael.peters@unibas.ch

⁴ Dep. of Mathematics, Seminar für Angewandte Mathematik, Rämistrasse 101, 8092 Zürich, christoph.schwab@sam.math.ethz.ch

Numerical Solution of Boundary Value Problems on Domains with a Thin Layer of Random Thickness

Helmut Harbrecht¹ Marc Dambrine² Isabelle Greff³ Benedicte Puig⁴

The present talk is dedicated to the numerical solution of boundary value problems on domains with a thin layer of different conductivity and of random thickness. By changing the boundary condition, the boundary value problem given on the random domain can be transformed into a boundary value problem on a fixed domain. The randomness is then contained in the coefficients of the new boundary condition. The solution of this new boundary value problem approximates the original solution with leading order in the scale parameter ε of the layer's thickness. With the help of the Karhunen-Loève expansion, we transform this random boundary value problem into a deterministic, parametric one with a possibly high-dimensional parameter \mathbf{y} . Based on the decay of the random fluctuations of the layer's thickness, we prove rates of decay of the derivatives of the random solution with respect to this parameter \mathbf{y} which are robust in the scale parameter ε . Numerical results validate our theoretical findings.

References:

- [1] M. Dambrine, I. Greff, H. Harbrecht, and B. Puig. Numerical solution of the Poisson equation with a thin layer of random thickness. *SIAM J. Numer. Anal.*, 54(2):921–941, 2016.
- [2] M. Dambrine, I. Greff, H. Harbrecht, and B. Puig. Numerical solution of the homogeneous Neumann boundary value problem on domains with a thin layer of random thickness. Preprint 2016-06, Mathematisches Institut, Universität Basel, Switzerland, 2016.

¹ University of Basel, Department of Mathematics and Computer Science, Basel, Switzerland,
`helmut.harbrecht@unibas.ch`

² Laboratoire de Mathématiques Appliquées de Pau, Université de Pau et des Pays de l'Adour, Pau, France,
`marc.dambrine@univ-pau.fr`

³ Laboratoire de Mathématiques Appliquées de Pau, Université de Pau et des Pays de l'Adour, Pau, France,
`isabelle.greff@univ-pau.fr`

⁴ Laboratoire de Mathématiques Appliquées de Pau, Université de Pau et des Pays de l'Adour, Pau, France,
`benedicte.puig@univ-pau.fr`

Least-Squares Methods for Elasticity with Weakly Imposed Symmetry

Fleurianne Bertrand¹ Z. Cai² E.Y. Park³ G. Starke⁴

The related physical equations of linear elasticity are the equilibrium equation and the constitutive equation, which expresses a relation between the stress and strain tensors. This is a first-order partial differential system such that a least squares method based on a stress-displacement formulation can be used whose corresponding finite element approximation does not preserve the symmetry of the stress [1]. In this talk, a new method based on the general Stokes equation and imposing symmetry condition on the stress is investigated by introducing the vorticity and applying the L2 norm least squares principle to the stress-displacement-vorticity system. The question of ellipticity due to the fact that all three variables are present in one equation is discussed, and the homogeneous least-squares functional is shown to be uniformly equivalent to the norm of $H(\operatorname{div}(\Omega))^d \times H^1(\Omega)^d \times L^2(\Omega)^{d_0}$, which implies optimal error estimates for its finite element subspaces. Further, the supercloseness of the least squares approximation to the standard mixed finite element approximations arising from the Hellinger-Reissner principle with reduced symmetry [2], is studied. This implies that the favourable conservation properties of the dual-based mixed methods and the inherent error control of the least squares method are combined. Additionally, a closer look will be taken at the error that appears using this formulation on domains with curved boundaries approximated by a triangulation [3]. In the higher-order case, parametric Raviart-Thomas finite elements are employed to this end. Finally, it is shown that an optimal order of convergence is achieved and illustrated numerically on a test example.

References:

- [1] Z. Cai, G. Starke. Least squares methods for linear elasticity. *SIAM J. Numer. Anal.* 42 (2004): 826-842
- [2] D. Boffi, F. Brezzi, and M. Fortin. *Mixed Finite Element Methods and Applications*. Springer-Verlag, Heidelberg, 2013. [Chp. 9]
- [3] F. Bertrand, S. Münzenmaier, and G. Starke. First-Order System Least Squares on Curved Boundaries: Lowest-Order Raviart-Thomas Elements. *SIAM J. Numer. Anal.* 52.2 (2014): 880-894.

¹ Universität Duisburg-Essen, Fakultät für Mathematik, Essen, Germany,
fleurianne.bertrand@uni-due.de

² Department of Mathematics, Purdue University, United States,
caiz@purdue.edu

³ Department of Mathematics, Purdue University, United States,
park296@purdue.edu

⁴ Fakultät für Mathematik, Universität Duisburg-Essen,
gerhard.starke@uni-due.de

Symmetric Mixed Formulation for Reinforced Elastomers

Hansjörg Schmidt¹ Arnd Meyer²

Elastomers are used in many applications like air suspensions and belts, like drive-, timing- or conveyor-belts. Reinforcing them with fibres or twines improve their stiffness in a wanted direction. Furthermore, these fibre-reinforced elastomers can undergo large deformations without failure. Unfortunately, the simulation of this materials is rather hard, due to the low shear modulus of the elastomers in contrast to the high bulk modulus and the high fibre-stiffness. In more detail, in the corresponding Newton-system the low shear modulus is multiplied with the positive definite part and the other two high values are multiplied with some low-rank parts. Of course, mixed finite element methods can be used to tackle the (almost)-incompressible behaviour of elastomers. Additionally, we discuss how a mixed formulation for the (almost)-inextensible behaviour of the fibres can be achieved. Further, we present an easy way to achieve a symmetric mixed formulation Newton-system, from the substitution of variables for our specific energy density.

¹ TU Chemnitz, Fakultät für Mathematik, Chemnitz, Germany,
hanss@hrz.tu-chemnitz.de

² TU Chemnitz, Fakultät für Mathematik,
a.meyer@mathematik.tu-chemnitz.de

Efficient Simulation of Short Fibre Reinforced Composites

Rolf Springer¹ Arnd Meyer²

Lightweight structures became more and more important over the last years. One special class of such structures are short fibre reinforced composites, produced by injection moulding. To avoid expensive experiments for testing the mechanical behaviour of these composites proper material models are needed. Thereby, the stochastic nature of the fibre orientation is the main problem.

In this talk we look onto the simulation of such materials in a linear thermoelastic setting. So, we use the stress-strain relation

$$\sigma = \mathfrak{C} : (\varepsilon - (\theta - \theta_0)\mathbf{T}),$$

with a fourth order material tensor \mathfrak{C} , a second order thermal expansion tensor \mathbf{T} , the temperature difference $(\theta - \theta_0)$, and the second order linearised strain tensor ε . The temperature field θ within this equation is described by

$$-\nabla \cdot (\kappa \cdot \nabla \theta) = \Theta,$$

whereas θ_0 describes a reference field. In the last equation κ describes the heat conduction and is a symmetric second order tensor.

In both equations the material properties (κ , \mathbf{T} , and \mathfrak{C}) depend on the stochastic fibre orientation. We will present a way how this problem can be treated for the computation of the arising stresses and will present some numerical results.

¹ TU Chemnitz, Mathematics, Chemnitz, Germany,
rolf.springer@mathematik.tu-chemnitz.de

² TU Chemnitz, Mathematics, Chemnitz, Germany,
arnd.meyer@mathematik.tu-chemnitz.de

Locking in Variations of the Mindlin-Reissner Plate Model

Michael Weise¹

The Mindlin–Reissner plate model is widely used for the elastic deformation simulation of moderately thick plates. Shear locking occurs in the case of thin plates, which means slow convergence with respect to the mesh size. The Kirchhoff plate model does not show locking effects, but is valid only for thin plates. One would like to have a method suitable for both thick and thin plates.

Different approaches are known to deal with the shear locking in the Mindlin–Reissner model. One possible way is a hierarchical deformation ansatz combining the Kirchhoff and Mindlin–Reissner models. We investigate several such hierarchical methods with respect to the severeness of locking and the performance of the preconditioned conjugate gradient method used to solve the resulting finite element system in our talk.

¹ TU Chemnitz, Mathematics, Chemnitz,
michael.weise@mathematik.tu-chemnitz.de

Non-Conforming Grids for Flexible Discretization with Applications to Computational Acoustics

Manfred Kaltenbacher¹

We investigate flexible discretization techniques for the approximate solution of wave propagation problems. In order to keep as much flexibility as possible, we use independently generated grids which are well suited for approximating the solution of decoupled local sub-problems in each subdomain. Therefore, we have to deal with the situation of non-conforming grids appearing at the common interface of two subdomains. Special care has to be taken in order to define and implement the appropriate discrete coupling operators. We apply the Finite Element (FE) method and use three approaches to handle non-conforming grids: (1) Mortar coupling (2) Nitsche type mortaring and (3) Hybrid Nitsche type mortaring. In the first approach we guarantee the strong coupling of the numerical flux (normal derivative of the acoustic pressure) by introducing a Lagrange multiplier and coupling of the acoustic pressure in a weak sense. The Nitsche's type coupling does not need the additional Lagrange multiplier and handles the coupling by symmetrizing the bilinear form and adding a special jump term. Thereby, the method can be also interpreted as an IP-DG (Internal Penalty - Discontinuous Galerkin) at the interface. Finally, the hybrid Nitsche type mortaring introduces an additional hybrid variable to allow for an efficient decoupling of subdomains. Furthermore, the degrees of freedom of the hybrid variable are defined globally with respect to the geometry and represented by means of basis splines. The interface geometry is given by NURBS (Non-uniform Rational Basis Splines) allowing for an exact description of curved interfaces, an aspect incorporated from Isogeometric Analysis.

We will compare the proposed methods, discuss their advantages and disadvantages and will apply them to practical examples in computational acoustics.

¹ Institute of Mechanics and Mechatronics, TU Wien, Austria,
manfred.kaltenbacher@tu-wien.ac.at

Discontinuous Galerkin Isogeometric Analysis on Decompositions with Gaps and Overlaps

Ioannis Touloupoulos¹ Christoph Hofer² Ulrich Langer³

In the Isogeometric Analysis framework for treating realistic problems, it is usually necessary to decompose the domain into volumetric subdomains (patches). More precisely, we apply a segmentation technique for splitting the initial domain into simpler subdomains and then we define the corresponding control nets of the subdomains that are used for constructing the parametrizations of the subdomains. Usually, we obtain compatible parametrizations of the subdomains, meaning that using a relative coarse control mesh, the parameterizations of the adjoining subdomain interfaces are identical. However, this is not always the case. Due to an incorrect segmentation procedure, we can lead to non-compatible parametrizations of the geometry, meaning that the parametrized interfaces of adjoining subdomains are not identical. The result of this phenomenon is the creation of overlapping regions or gap regions between adjacent subdomains. It is clear that, we can not apply directly the dGIGA methods which have been proposed so far in the literature and are referred to matching interface parametrizations. In this talk, we will present a discontinuous Galerkin Isogeometric Analysis method applied on decompositions, where gap and overlapping regions can appear. We apply a multi-patch approach and derive suitable numerical fluxes on the boundaries of overlapping and gap regions, using the interior subdomain solutions, i.e., the solution on points which are not located on the overlaps and on gaps. These fluxes are used in order to couple the local patch-wise discrete problems. The ideas are illustrated on a model diffusion problem with discontinuous diffusion coefficients. We develop a rigorous theoretical framework for the proposed method clarifying the influence of the gap/overlapping region size onto the convergence rate of the method. The theoretical estimates are supported by numerical examples in two- and three-dimensional computational domains. We gratefully acknowledge the financial support of this research work by the Austrian Science Fund (FWF) under the grant NFN S117-03.

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¹ Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austrian Academy of Sciences, Linz, AUSTRIA,
ioannis.touloupoulos@oeaw.ac.at

² RICAM, Austrian Academy of Sciences,
christoph.hofer@ricam.oeaw.ac.at

³ RICAM, Austrian Academy of Sciences,
ulrich.langer@ricam.oeaw.ac.at

Fast Realization of Eikonal Equation Solvers on Tablet Devices

Gundolf Haase¹ Daniel Ganellari²

The Eikonal equation describes the arrival times of an expanding potential in a given domain Ω . This equation can be considered as a reduced model of the bidomain equations describing the time dependent development of the intercellular potential in a heart simulation after some excitation.

$$\begin{aligned} |\nabla u(x)| &= F(x), & x \in \Omega \\ u &= 0, & x \in \Gamma \subset \partial\Omega \end{aligned}$$

The Eikonal equation is a special case of the Hamilton-Jacobi partial differential equations (PDEs), encountered in problems of wave propagation.

The solution algorithm bases on the Fast Iteration Method (FIM) algorithm from [1] with special focus on tetrahedral elements [2]. We will present our achievements on implementing the code with OpenMP, MPI and CUDA on a wide variation of hardware ranging from cluster computing with very recent ARMv9 processors to tablets with the Tegra X1 processor. A special focus will be on the domain decomposition approach for parallelizing FIM and on a very sophisticated implementation such that the memory footprint will be significantly reduced in comparison to [Fu, Kirby and Whitaker, 2013].

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¹ University of Graz, Institute for Mathematics and Scientific Computing, Graz, Austria,
gundolf.haase@uni-graz.at

² ganellari.d@gmail.com

Weighted Finite Element Method for the Elasticity Problem with Singularity

Viktor Rukavishnikov¹

We consider the two-dimensional elasticity problem with a singularity caused by the presence of a reentrant corner on the domain boundary. For this problem, the notion of the R_ν -generalized solution is introduced and the corresponding definition is used to construct a scheme of the weighted finite-element method (FEM). The proposed method provides a first-order convergence of the approximate solution to the exact one with respect to the mesh step in the $\mathbf{W}_{2,\nu}^1(\Omega)$ -norm. The convergence rate does not depend on the size of the angle and kind of the boundary conditions imposed on its sides. This statement is illustrated by the results of numerical experiments. For the model problems, the dependence of the rate of convergence of the approximate solution to the exact R_ν -generalized solution with respect to the regularization parameters δ and ν is investigated.

¹ Computing Center, Far-Eastern Branch, Russian Academy of Sciences, Hard of Laboratory "Mathematical Modeling in Physical and Engineering", Khabarovsk, Russia,
vark0102@mail.ru

Energy-Correction Method for Parabolic Problems

Piotr Swierczynski¹ Barbara Wohlmuth²

Let $\Omega \subset \mathbb{R}^2$ be a bounded polygonal domain with a re-entrant corner, i.e. corner with an angle $\Theta > \pi$. In this talk we consider a model linear heat equation and investigate the behaviour of its piecewise linear finite element approximation in space. Its spatial convergence order is lower due to reduced regularity in the presence of the re-entrant corner. Recently, an effective method of recovering full second-order convergence for elliptic equations on domains with re-entrant corners, when measured in weighted L_2 and H^1 norms, known as energy-correction, has been proposed. This method is based on a modification of a fixed number of entries in the system's stiffness matrix. We show how energy-correction method can be successfully applied to regain optimal convergence in weighted norms for parabolic problems. Moreover, we propose a post-processing strategy yielding optimal convergence order in standard Sobolev norms.

Standard discretisation approach involving graded meshes results in a very restrictive form of a CFL condition, making the use of explicit time stepping practically impossible. On the other hand, energy-corrected finite element can be used on uniform meshes allowing for application of explicit time stepping scheme with relatively large time steps. This, combined with mass-lumping strategy, leads to a very efficient discretisation of parabolic problems, where at each time step only one vector multiplication with a scaled stiffness matrix needs to be performed.

All theoretical results are confirmed by numerical test.

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¹ Technische Universitat Munchen, Lehrstuhl fur Numerische Mathematik, Garching bei Munchen, Germany, piotr.swierczynski@ma.tum.de

² Technische Universitat Munchen, Lehrstuhl fur Numerische Mathematik, Garching bei Munchen, Germany, wohlmuth@ma.tum.de

Extended Finite Element Methods Dealing with Singularities on Meshes of Combined Dimensions

Pavel Exner¹ Jan Březina²

Our research is aimed at modelling of groundwater flow with finite element method on meshes of combined dimensions. General problems of these models (also in many other applications) is disability to capture small-scale phenomena in large domains and creation of suitable conforming meshes in real world situations. The extended finite element method (XFEM) provides a way to overcome these problems. However, its usage in models with combined dimensions is still developing and has many open questions. The main goal is to propose and implement a proper XFEM enrichment in a model of groundwater flow, governed by Darcy's law, to enable coupling between dimensions on non-conforming meshes. In particular, we are interested in 0D and 1D singularities in 2D and 3D, respectively.

We study different enrichment methods for pressure in a steady well-aquifer problem. The finite element space is enriched with an analytical solution to a local Laplace problem with a point source. We develop accurate quadrature rules for integration over enriched elements to obtain the optimal order of convergence. We also investigate a reasonable choice of the enriched area.

Further, we aim our effort at usage of the XFEM in the software Flow123d, a simulator of underground water flow and solute transport on meshes of combined dimension. Since the knowledge of an accurate velocity field is of a great importance in the transport equation, the mixed-hybrid form of the problem is used with the lowest order Raviart-Thomas finite elements. We are interested in finding a suitable enrichment both for pressure and velocity, however, the satisfaction of the inf-sup condition in this case is in question.

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¹ Technical University of Liberec, Studentska 1402/2, 46117 Liberec, Czech Republic,
Technische Universität München, Lehrstuhl für Numerische Mathematik, Garching bei München, Germany,
pavelexner@gmail.com

² Technical University of Liberec, Studentská 1402/2, 46117 Liberec, Czech Republic,
jan.brezina@tul.cz

Stress Reconstruction for the Nonconforming P2 Finite Element Method and A Posteriori Error Estimation

Marcel Moldenhauer¹ Gerhard Starke²

We want to follow the framework given in [1] and present an a posteriori error estimator for the nonconforming P2 finite element method of the linear elasticity problem based on a nonsymmetric $H(\text{div})$ -conforming approximation of the stress tensor.

In [2] a nonconforming P2 finite element method was used with success to reconstruct an $H(\text{div})$ -conforming flux with application to a posteriori error estimation. We want to study the positive characteristics of nonconforming P2 elements in the framework of [1] and confirm the quality of our stress reconstruction and effectiveness of our error estimator with numerical examination of the Cooks membran problem. See also [3].

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¹ Fakultät für Mathematik, Universität Duisburg-Essen, , Thea-Leymann-Straße 9, 45217 Essen, Germany, marcel.moldenhauer@uni-due.de

² Fakultät für Mathematik, Universität Duisburg-Essen, , Thea-Leymann-Straße 9, 45217 Essen, Germany, gerhard.starke@uni-due.de

Error Estimates for the Numerical Approximation of Sparse Parabolic Control Problems

Eduardo Casas¹

In this talk we consider the following optimal control problem

$$(P) \quad \min_{u \in L^\infty(Q)} J(u) = F(u) + \mu j(u),$$

where

$$F(u) = \frac{1}{2} \int_Q (y_u(x, t) - y_d(x, t))^2 dx dt + \frac{\nu}{2} \int_Q u^2(x, t) dx dt,$$

$$j(u) = \int_\Omega \left(\int_0^T u^2(x, t) dt \right)^{1/2} dx,$$

and y_u is the state associated to the control u , solution of

$$\begin{cases} \partial_t y - \Delta y + a(x, t, y) = u & \text{in } Q = \Omega \times (0, T), \\ y = 0 & \text{on } \Sigma = \Gamma \times (0, T), \\ y(0) = y_0 & \text{in } \Omega. \end{cases}$$

We prove the existence of at least one solution to problem (P), and establish the first and second order optimality conditions. From the first order optimality conditions we deduce the spatial sparsity of the optimal controls as well as some regularity properties. Finally, we define a finite-element based approximation of the control problem and, assuming a second order sufficient optimality condition, we prove L^2 error estimates for the difference between the locally optimal controls and their discrete approximations.

¹ Dpto. de Matemática Aplicada y Ciencias de la Computación, E.T.S.I. Industriales y de Telecomunicación, Universidad de Cantabria, 39005 Santander, Spain,
eduardo.casas@unican.es

Piecewise Linear Control Functions for Semilinear Optimal Control Problems with Directional Sparsity

Arnd Rösch¹ Eduardo Casas² Mariano Mateos³

This talk is a continuation of the plenary talk given by Eduardo Casas. We discuss the Finite Element discretization of a semilinear parabolic optimal control problem with a directional sparsity functional by piecewise linear control functions. A straight forward discretization of the optimal control problem would be a bad choice for two reasons: The directional sparsity is not preserved and the obtained a priori error estimates have a low order. Therefore we replace the two control norms by appropriate discrete norms. The first task is to show the directional sparsity for the discrete controls. A second topic is the derivation of a priori error estimates for the optimal control and the optimal state.

This talk is a joint work with Eduardo Casas and Mariano Mateos.

¹ Universität Duisburg-Essen, Fakultät für Mathematik, Essen, Germany,
arnd.roesch@uni-due.de

² Departamento de Matematica Aplicada y Ciencias de la Computacion, E.T.S.I. Industriales y de Telecomunicacion, Universidad de Cantabria, 39005 Santander, Spain,
eduardo.casas@unican.es

³ Departamento de Matematicas, E.P.I. Gijon, Universidad de Oviedo, Campus de Gijon, 33203 Gijon, Spain,
mmateos@uniovi.es

Optimal Control of Mechanical Damage Processes - Numerical Simulation and Optimization

Marita Holtmannspötter¹ Christian Meyer² Arnd Rösch³

We study an optimal control problem of a new model for mechanical damage processes. In general the corresponding control-to-state mapping is not differentiable. Therefore we focus on effects arising from this property. We present a new numerical approach for the simulation of the system and the optimal control problem and provide test results which support our approach.

¹ University of Duisburg-Essen, Faculty of Mathematics, Essen, Germany,
marita.holtmannspoetter@uni-due.de

² TU Dortmund,
christian2.meyer@tu-dortmund.de

³ University of Duisburg-Essen,
arnd.roesch@uni-due.de

Fast Solvers for hp -FEM Discretized PDE-Constrained Optimization Problems

Sven Beuchler¹ Katharina Hofer²

In this talk, we investigate the minimization of a quadratic functional

$$J(u, y) = \frac{1}{2} \int_{\Omega} (y - y_d)^2 dx + \frac{\alpha}{2} \int_{\Omega} u^2 dx$$

governed the boundary value problem

$$-\Delta y + y = u \quad \in \Omega$$

with some boundary conditions for y . This problem is discretized by hp -finite elements. The main focus of this talk is the development of efficient solution methods for the corresponding system of linear algebraic equations. We consider the solvers:

- a conjugate gradient method in a special inner product, following Schöberl/Zulehner
- the minimal residual method (MINRES)

In both methods, efficient preconditioners for mass and stiffness matrix accelerate the convergence speed of the iterative method. This contribution presents overlapping hp -FEM preconditioners for mass and stiffness matrix.

¹ Uni Bonn, INS, Bonn, Bonn,
beuchler@ins.uni-bonn.de

² INS, Uni Bonn,
hofer@ins.uni-bonn.de

Discretization of Control Constrained Optimal Control Problems with Mass Lumping

Gerd Wachsmuth¹ Arnd Rösch²

In this talk we consider the optimal control of an elliptic PDE subject to pointwise control constraints. We discretize the problem with higher order finite elements in order to achieve fast convergence with respect to the mesh width h .

We propose a discretization of the optimal control problem which uses lumping for the mass matrices. This scheme has the advantage that “optimization” and “discretization” commute. We provide a-priori error estimates. In the case of P^2 finite elements, we achieve the order h^2 for the L^2 error in the state and the adjoint state on polygonal convex domains. We also obtain convergence order h^2 for the optimal control in the nodes of the discretization and for a post-processed control. The order can be improved to h_{global}^3 by a local refinement at the boundary of the active set.

The theoretical findings are confirmed by numerical experiments.

¹ TU Chemnitz, Fakultät für Mathematik, Chemnitz, Germany,
gerd.wachsmuth@mathematik.tu-chemnitz.de

² Universität Duisburg-Essen, Fakultät für Mathematik,
arnd.roesch@uni-due.de

Discretization Error Estimates for Dirichlet Control Problems in Polygonal Domains

Thomas Apel¹ Mariano Mateos² Johannes Pfefferer³ Arnd Rösch⁴

In this talk we discuss convergence results for finite element discretized Dirichlet control problems in polygonal domains. We investigate unconstrained as well as control constrained problems. In both cases we discretize the state and the control by piecewise linear and continuous functions. The error estimates mainly depend on the size of the interior angles but also on the presence of control constraints and the structure of the underlying mesh. For instance, considering non-convex domains, the convergence rates of the discrete optimal controls in the unconstrained case can even be worse than in the control constrained case.

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¹ Universität der Bundeswehr München, Institut für Mathematik und Bauinformatik, Neubiberg, Germany, thomas.apel@unibw.de

² Departamento de Matemáticas, Universidad de Oviedo, Spain, mmateos@uniovi.es

³ Lehrstuhl für Optimalsteuerung, Technische Universität München, Germany, pfefferer@ma.tum.de

⁴ Fakultät für Mathematik, Universität Duisburg-Essen, Germany, Arnd.Roesch@uni-due.de

Weighted Finite Element Error Estimates on Boundary Concentrated Meshes with Application to Dirichlet Control

Max Winkler¹ Johannes Pfefferer²

In this talk we present new error estimates in special $L^2(\Omega)$ -norms containing the distance towards the boundary as weight function. These error estimates are of interest when deriving a priori estimates for the approximate normal derivative of elliptic equations, and, the application that we want to focus on, for the numerical approximation of Dirichlet control problems where the control variable corresponds to the normal derivative of some adjoint variable. Our aim is to improve the accuracy of the numerical approximation by refining the computational meshes towards the boundary of the underlying domain.

We restrict our considerations to the Poisson equation with Dirichlet boundary conditions in convex polygonal domains. As the vertices in the domain cause also singularities we have to deal with regularity results in weighted Sobolev spaces containing the distance to the vertices and the boundary as weight functions. To prove error estimates we extend a technique that is used to prove local maximum norm estimates, more precisely, we apply a combination of dyadic decompositions with respect to the element boundary and the vertices of the computational domain to carve out both types of singularities. As a main result we show that the convergence rate of almost two (up to logarithmic factors) can be achieved also in weighted $L^2(\Omega)$ -norms provided that the vertex singularities are mild enough. As a by-product we will observe that singularities corresponding to vertices having interior angle less than 120° do not lower the convergence rates. If there are vertices having a larger opening angle we prove lower convergence rates that depend solely on those angles. The predicted behavior is confirmed in our numerical experiments.

¹ Universität der Bundeswehr München, Institut für Mathematik und Bauinformatik, Neubiberg, Germany,
max.winkler@unibw.de

² Technische Universität München,
pfefferer@ma.tum.de

List of Participants

List of participants

Surname, first name	Abstr.	from	e-mail
Apel, Thomas	[34]	Neubiberg	thomas.apel@unibw.de
Bause, Markus	[9]	Hamburg	bause@hsu-hh.de
Becher, Simon	[12]	Dresden	Simon.Becher@tu-dresden.de
Bertrand, Fleurianne	[18]	Essen	fleurianne.bertrand@uni-due.de
Beuchler, Sven	[32]	Bonn	beuchler@ins.uni-bonn.de
Casas, Eduardo	[29]	Santander	eduardo.casas@unican.es
Dötz, Jürgen	[16]	Basel	juergen.doelz@unibas.ch
Egger, Herbert	[15]	Darmstadt	egger@mathematik.tu-darmstadt.de
Ernst, Oliver		Chemnitz	oliver.ernst@mathematik.tu-chemnitz.de
Exner, Pavel	[27]	Garching/Munich	pavelexner@gmail.com
Haase, Gundolf	[24]	Graz	gundolf.haase@uni-graz.at
Harbrecht, Helmut	[17]	Basel	helmut.harbrecht@unibas.ch
Herzog, Roland		Chemnitz	roland.herzog@mathematik.tu-chemnitz.de
Holtmannspötter, Marita	[31]	Essen	marita.holtmannspoetter@uni-due.de
Jung, Michael		Dresden	mjung@informatik.htw-dresden.de
Linß, Torsten		Hagen	torsten.linss@fernuni-hagen.de
Kaltenbacher, Manfred	[22]	Vienna	manfred.kaltenbacher@tuwien.ac.at
Lube, Gert	[11]	Goettingen	lube@math.uni-goettingen.de
Ludwig, Lars		Dresden	lars.ludwig@tu-dresden.de
Matthies, Gunar		Dresden	gunar.matthies@tu-dresden.de
Meszmer, Peter		Chemnitz	peter.meszmer@etit.tu-chemnitz.de
Meyer, Arnd		Chemnitz	a.meyer@mathematik.tu-chemnitz.de
Moldenhauer, Marcel	[28]	Essen	marcel.moldenhauer@gmail.com
Ospald, Felix		Chemnitz	felix.ospald@gmail.com

Surname, first name	Abstr.	from	e-mail
Pester , Matthias		Chemnitz	pester@mathematik.tu-chemnitz.de
Roskovec , Filip		Prague 8	roskovec@gmail.com
Rukavishnikov , Viktor	[25]	Khabarovsk	vark0102@mail.ru
Rösch , Arnd	[30]	Essen	arnd.roesch@uni-due.de
Schieweck , Friedhelm	[10]	Magdeburg	schiewec@ovgu.de
Schmidt , Hansjörg	[19]	Chemnitz	hanss@hrz.tu-chemnitz.de
Simon , Kristin	[14]	Magdeburg	Kristin.Simon@ovgu.de
Skrzypacz , Piotr	[13]	Astana	piotr.skrzypacz@nu.edu.kz
Springer , Rolf	[20]	Chemnitz	rolf.springer@mathematik.tu-chemnitz.de
Sprungk , Björn		Chemnitz	rolf.springer@mathematik.tu-chemnitz.de
Swierczynski , Piotr	[26]	Chemnitz	bjoern.sprungk@mathematik.tu-chemnitz.de
Toulopoulos , Ioannis	[23]	Garching/Munich	piotr.swierczynski@ma.tum.de
Wachsmuth , Gerd	[33]	Linz	ioannis.toulopoulos@oeaw.ac.at
Weise , Michael	[21]	Chemnitz	gerd.wachsmuth@mathematik.tu-chemnitz.de
Wihler , Thomas	[8]	Bern	michael.weise@mathematik.tu-chemnitz.de
Winkler , Max	[35]	Neubiberg	thomas.wihler@math.unibe.ch
			max.winkler@unibw.de

Internet access

The hotel offers free internet access. Wireless LAN is available in all rooms. Access details can be obtained from the hotel reception.

Food

The conference fee includes:

- lunch on all three days of the symposium (one soft drink included),
- tea, coffee, soft drinks and snacks during breaks,
- the conference dinner on Monday.

For participants staying at the conference hotel there is a breakfast buffet from 6 am up to 10 am.

Recreation

The hotel offers a fitness room with sauna for free. Moreover, the "Café Moskau" next door offers billard and snooker tables.

Conference Dinner

The conference dinner takes place on Monday at 8 pm in the restaurant Max-Louis.

Drinks are at your own expense.

We will meet at 7:20 pm in front of the "Hotel an der Oper" and walk together to the restaurant. If you would like to go on your own, the address of the restaurant is:

Schönherrstraße 8, 09113 Chemnitz

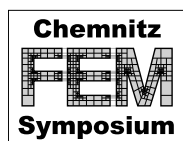
Excursion

The excursion will take place on Tuesday. We will walk to the Chemnitz Museum of Industry for a guided tour. Therefore, we will meet at 2 pm in front of the "Hotel an der Oper". The distance is approximately 3 km. After the visit of the museum we will have a break at the nearby coffee house "Kohlebunker" (at your own expense).

If you want to go by bus or tram you can use the bus 51 or tram 4 or 6. The bus/tram leaves from station "Theaterplatz" (in front of the hotel) to central station. Then change to bus 23 or tram 1 and get out at station "Industriemuseum". The tram departs every 10 minutes and the bus every 30 minutes. Travelling time is about 15 minutes.

The address of the Chemnitz Museum of Industry is:

Zwickauer Str. 119, 09112 Chemnitz.



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Technische Universität Chemnitz
09107 Chemnitz
www.tu-chemnitz.de