

# Non-Conforming Galerkin Finite Element Methods for Local Absorbing Boundary Conditions of Higher Order

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Local absorbing boundary conditions are used to mimick the solution in presence of an infinite exterior in diffusion problems or time-harmonic scattering problems, in highly conducting bodies or thin layers. We consider Dirichlet-to-Neumann boundary conditions involving higher tangential derivatives (see an analysis in K. Schmidt and C. Heier, ESAIM Math. Model. Numer. Anal., 49(1): 257–273, 2015). If only second derivatives are present, *i.e.*, for the Neumann, Robin and Wenttzel conditions, and the boundary is smooth enough, we can incorporate the condition in usual piecewise continuous finite element methods. For higher derivatives trial and test functions with higher continuity (at least) along the boundary or auxilliary unknowns may be used. We propose as an alternative nonconforming interior penalty finite element methods for usual continuous finite element spaces in additional terms on the nodes of the boundary appear. For fourth order PDEs a similar approach has been introduced in S. Brenner and L.-Y. Sung, J. Sci. Comput. 22-23, 84–118, 2005. We will present well-posedness results and *a-priori*  $h$ -convergence error estimates for uniform polynomial degrees. The theoretical convergence results are validated by a series of numerical experiments.

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