TECHNISCHE UNIVERSITÄT
CHEMNITZ

Fakultät für Mathematik

Chemnitz FEM-Symposium 2015

Chemnitz
FEM
Symposium

Programme

Collection of abstracts

List of participants

Chemnitz, September 28 - 30, 2015

Scientific Topics:

The symposium is devoted to all aspects of finite elements and wavelet methods in partial differential equations.

The topics include (but are not limited to):

- adaptive methods,
- parallel implementation,
- high order methods.

This year we particularly encourage talks on:

- adaptivity and error estimation,
- mixed and least quares finite elements,
- multiscale methods.

Invited Speakers:

Martin Vohralík (Inria Paris-Rocquencourt)

Gerhard Starke (Universität Duisburg-Essen)

Daniel Peterseim (Universität Bonn)

Assyr Abdulle (EPF Lausanne)

Conference Venue:

Center Hotel "Alte Spinnerei"
Chemnitzer Straße 89-91
09217 Burgstädt, Germany
http://www.hotel-altespinnerei.de

Scientific Committee:

Th. Apel (München), S. Beuchler (Bonn), O. Ernst (Chemnitz), G. Haase (Graz), H. Harbrecht (Basel), R. Herzog (Chemnitz), M. Jung (Dresden), U. Langer (Linz), A. Meyer (Chemnitz), A. Rösch (Duisburg), O. Steinbach (Graz)

Organising Committee:

R. Springer, B. Sprungk, H. Schmidt, M. Pester, A.-K. Glanzberg, K. Seidel

https://www.tu-chemnitz.de/mathematik/fem-symposium/

Programme

Programme for Monday, September 28, 2015

9:00	**Opening**	*Room*: 7

Mixed FE
Chair: A. Meyer — *Room*: 7

10:45 *Tea and coffee break*

	Mixed FE *Chair*: G. Starke — *Room*: 7	**Solvers** *Chair*: A. Linke — *Room*: 8
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11:40	**Christian Merdon** . . . . . 12 A Modified Pressure-Robust 2nd-Order Finite Element Method for Navier-Stokes Discretisations.	**Bülent Karasözen** . . . . . 16 Space-Time Adaptive Discontinuous Galerkin Methods for Semilinear Diffusion-Convection-Reaction Equations.
12:05	**Fleurianne Bertrand** . . . . . 13 Raviart-Thomas Elements on Curved Domains.	**Lennard Kamenski** . . . . . 17 Conditioning of Linear Finite Element Equations with Arbitrary Anisotropic Grids.
12:30	**Clemens Bartsch** . . . . . 14 An Assessment of Solvers for Saddle Point Problems Emerging from the Incompressible Navier-Stokes Equations.	**Huidong Yang** . . . . . 18 Numerical Simulation of the Interface Moving and Growing Problems Using Small Mesh Deformation.

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Programme for Monday, September 28, 2015 – continued

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14:55	**Hansjörg Schmidt** ... 20 Viscoelasticity with Internal State Variables at Large Strain Deformations.	**Vera Bommer** ... 24 Numerical Analysis for the Optimal Control of the Full Time-Dependent Maxwell Equations.
15:20	**Rainer Hartmann** ... 21 Full Vectorial Finite Element Beam Propagation Method for Simulation of Depolarization in Laser Amplifiers.	**Constantin Christof** ... 25 A-priori L^p-Error Analysis for the Obstacle Problem.
15:45	**Rolf Springer** ... 22 Efficient Simulation of Short Fibre Reinforced Composites.	**Kersten Schmidt** ... 26 Non-Conforming Galerkin Finite Element Methods for Local Absorbing Boundary Conditions of Higher Order.
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Lunch

Collection of abstracts

Stress Approximation for Elasticity Computations by First-Order System Finite Element Methods

Gerhard Starke[1] Benjamin Müller[2]

In this talk, different finite element methods are examined regarding their ability to produce accurate approximations of the stresses associated with elasticity problems. Such approximations are of interest in many applications in solid mechanics since large local stresses may cause inelastic material behavior or failure. Accurate approximations of surface traction forces also rely on the good resolution of the stress components. The focus of this presentation is on elastic deformations starting from linear elasticity and then turning to hyperelastic models involving geometrical and material nonlinearities. Of particular interest are approaches which remain uniformly accurate in the limit of incompressible materials.

The standard way to perform elasticity computations is based on the representation of the displacement variable by suitable finite element spaces which are augmented, in the incompressible regime, by an additional pressure variable. From these finite element approximations, accurate stresses can be reconstructed in a localizable post-processing step. An alternative approach consists in the use of variational formulations involving the stress as an independent variable which is approximated directly in suitable $H(div)$-conforming finite element spaces. Such approaches may either be of saddle-point or of least-squares type and relations between these two will be investigated in detail.

The approximations obtained from the stress-based finite element approaches will be compared computationally with those obtained from a reconstruction procedure. For all of the above approaches, stress approximations in Raviart-Thomas spaces of lowest and next-to-lowest order will be produced. This is done for some two- and three-dimensional model problems in the linearly elastic as well as the hyperelastic setting including incompressible materials.

References:

[1] B. Müller, G. Starke, A. Schwarz, J. Schröder: A First-Order System Least Squares Method for Hyperelasticity. SIAM J. Sci. Comput. 36: B795-B816 (2014)

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Towards Gradient-Robust Mixed Methods for the Incompressible Navier-Stokes Equations

Alexander Linke[1]

Mixed methods for the incompressible Navier-Stokes equations are reviewed with respect to the discretization of the divergence constraint. Though the establishment of inf-sup stable mixed methods represents a milestone in the development of discretization theory for flow problems, many important questions are left open, and classical text books usually convey a wrong impression what are they best qualitatively possible results, which are achievable in the field. Especially, it will be shown that the construction of gradient-robust mixed methods, whose velocity error is pressure-independent, is rather easy, though this was thought to be nearly impossible for many years. Numerical examples will show that classical mixed methods deliver poor results, whenever large irrotational forces appear in the Navier-Stokes momentum balance, while gradient-robust mixed methods perform well.

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Robust Arbitrary Order Mixed Finite Element Methods for the Incompressible Stokes Equations with Pressure Independent Velocity Errors

Gunar Matthies[1] Alexander Linke[2] Lutz Tobiska[3]

Standard mixed finite element methods for the incompressible Navier–Stokes equations that relax the divergence constraint are not robust against large irrotational forces in the momentum balance and the velocity error depends on the continuous pressure. This robustness issue can be completely cured by using divergence-free mixed finite elements which deliver pressure-independent velocity error estimates. However, the construction of H^1-conforming, divergence-free mixed finite element methods is rather difficult. Instead, we present a novel approach for the construction of arbitrary order mixed finite element methods which deliver pressure-independent velocity errors. The approach does not change the trial functions but replaces discretely divergence-free test functions in some operators of the weak formulation by divergence-free ones. This modification is applied to inf-sup stable conforming and nonconforming mixed finite element methods of arbitrary order in two and three dimensions. Optimal estimates for the incompressible Stokes equations are proved for the H^1 and L^2 errors of the velocity and the L^2 error of the pressure. Moreover, both velocity errors are pressure-independent, demonstrating the improved robustness. Several numerical examples illustrate the results.

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On Mixed FE-Models for Variational Inequalities

Andrej Garanza[1] Suttmeier, Franz-Theo[2]

The presentation deepens our studies on the numerical FE-treatment of systems of partial differential equations, where the solution is subjected to inequality constraints.

Especially we focus on Lagrange-settings, which can be employed to handle the given constraints. In this way additional auxiliary variables are introduced which are determined simultaneously to the original primal solution within a so-called mixed system.

On this basis efficient solution processes for the mixed systems are constructed by eliminating inequality constraints yielding nonlinear equation systems. These can easily be solved by (non-smooth) Newton-type schemes.

Furthermore concepts for a posteriori error control are reviewed and refined.

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A Modified Pressure-Robust 2nd-Order Finite Element Method for Navier-Stokes Discretisations

Christian Merdon[1] Alexander Linke[2]

The Taylor-Hood finite element method can be enriched with bubble functions and so allows for piecewise linear pressure (without continuity) in the discretisation of the Navier-Stokes equations. This increases the stability of the method in presence of complicated pressures.

However, there still remains a pressure-dependence of the a priori velocity error estimates due to non divergence-free test functions. This can be remedied by a variational crime in the spirit of the references below that employs local reconstruction of the bubble functions into the Brezzi-Douglas-Marini elements.

Several numerical examples compare and illustrate the robustness of the modified method. In examples with complicated pressures the errors are significantly smaller than for the unstabilised method and much smaller than for the Taylor-Hood finite element method.

References:

[1] A. Linke, *On the role of the Helmholtz decomposition in mixed methods for incompressible flows and a new variational crime*, Comput. Methods Appl. Mech. Engrg. 268 (2014), 782–800.
[2] C. Brennecke, A. Linke, C. Merdon, J. Schöberl, *Optimal and pressure-independent L2 velocity error estimates for a modified Crouzeix-Raviart Stokes element with BDM reconstructions*, J. Comput. Math. 33 (2015), no. 2, 191–208.
[3] A. Linke, G. Matthies, L. Tobiska, *Robust arbitrary order mixed finite element methods for the incompressible Stokes equations*, WIAS Preprint 2027.

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Raviart-Thomas Elements on Curved Domains

Fleurianne Bertrand[1] Gerhard Starke[2] Steffen Münzenmeier[3]

Optimal order convergence of a first-order system least squares method using lowest-order Raviart-Thomas elements combined with linear conforming elements is presented for domains with curved boundaries. Parametric Raviart-Thomas elements are introduced in order to retain the optimal order of convergence in the higher-order case in combination with the isoparametric scalar elements. In particular, an estimate for the normal flux of the Raviart-Thomas elements on interpolated boundaries is derived in both cases. This is illustrated numerically for the Poisson problem on the unit disk. As an application of the analysis derived for the Poisson problem, boundary values of forces are estimated in the Stokes problem and the effect of interpolated interface condition for a stationary two-phase flow problem is then studied.

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An Assessment of Solvers for Saddle Point Problems Emerging from the Incompressible Navier-Stokes Equations

Clemens Bartsch[1] Naveed Ahmed[2] Volker John[3] Ulrich Wilbrandt[4]

This talk presents an assessment of the performance of different solvers for linear saddle point problems. In fluid dynamics large linear saddle point problems emerge from the linearization and discretization of the Navier–Stokes equations. Preconditioned Krylov subspace methods such as FGMRES are a popular choice to solve these large linear systems. We employ FGMRES with two different types of preconditioners: A geometric multigrid preconditioner with different implementations of Vanka type smoothers is compared with the least squares commutator preconditioner (LSC) of Elman et al. The LSC preconditioner has been modified recently as to include boundary conditions and comparisons of the LSC approach with geometric multigrid preconditioners are not available so far. We incorporate the original and the boundary corrected LSC preconditioner into our studies.
For comparison of these preconditioned iterative methods we also consider the direct solvers UMFPACK and PARDISO.
The solvers are appplied to several variants of the commmon benchmark example of a flow around a cylinder, in two and three dimensions as well as for the steady and the time-dependent Navier–Stokes equations.
The methods and examples are implemented in the in-house finite element software package MooN.

References:

[1] H. C. Elman and R. S. Tuminaro, Boundary conditions in approximate commutator preconditioners for the Navier-Stokes equations. ETNA 35: 257–280, 2009.
[2] H. C. Elman, V. E. Howle, J. Shadid, R. Shuttleworth and R. Tuminaro, Block preconditioners based on approximate commutators. SIAM J. Sci. Comput. 27(5): 1651–1668, 2006.
[3] V. John, High order finite element methods and multigrid solvers in a benchmark problem for the 3D Navier-Stokes equations. Int. J. Numer. Meth. Fluids 40: 775–798, 2002.
[4] V. John and G. Matthies, MooNMD - a program package based on mapped finite element methods. Comput Visual Sci. 6: 163–170, 2004.
[5] Y. Saad, A flexible inner-outer preconditioned GMRES algorithm. SIAM J. Sci. Comput. 14(2):461-469, 1993.
[6] M. Schäfer and S. Turek, The benchmark problem "Flow around a cylinder", in: Flow Simulation with High-Performance Computers II, vol. 52. Notes on Numerical Fluid Mechanics, Vieweg. 1996; 547–566.

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A-posteriori Estimates for the Rotational Pressure-Correction Projection Method

Andreas Brenner[1] Eberhard Bänsch[2]

A-posteriori error estimates for time discretization by the two-step backward differential formula method (BDF2) of the incompressible Stokes equations by pressure-correction methods in rotational form are presented. Moreover, rate optimality of the estimators are shown for velocity. Computational experiments confirm the theoretical results. A short introduction into projection schemes are given and the main tools for the a-posteriori analysis are presented.

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Space-Time Adaptive Discontinuous Galerkin Methods for Semilinear Diffusion-Convection-Reaction Equations

Bülent Karasözen[1] Murat Uzunca[2]

An accurate and efficient numerical resolution of interior/boundary layers in convective dominated problems is a challenge over many decades. In nonlinear stationary problems, the nonlinear reaction term produces sharp layers in addition to the spurious oscillations due to the convection. In the non-stationary case, the resolution of such layers is more critical since the nature of the sharp layers may vary as time progresses. In contrast to the stabilized continuous Galerkin finite element methods, discontinuous Galerkin (DG) methods produce stable discretizations without the need for stabilization parameters. Moreover, DG methods are better suited for adaptive strategies.

In this talk, we apply a time-space adaptive algorithm, which utilizes the elliptic reconstruction technique to be able to use the robust (in Péclet number) residual-based a posteriori error estimator, for the convection dominated reactive flow problems. We derive a posteriori error bounds in the $L^2(H^1)$ and $L^\infty(L^2)$-type norms using backward Euler in time and symmetric interior penalty Galerkin (SIPG) in space. We also investigate the influence of the flow field and surface tension on droplet breakup phenomena described by the non-local advective Allen-Cahn equation. In contrast to the reactive flow problems, where the velocity field is incompressible, in the advective Allen-Cahn equation the velocity field is expanding or contracting. After the discretization in time by the implicit Euler method, the resulting sequence of semi-linear elliptic equations are solved by the residual-based adaptive algorithm. Numerical results demonstrating the performance of the adaptive algorithm will be presented for convection dominated problems in reactive flow equations and non-local advective Allen-Cahn equation.

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Conditioning of Linear Finite Element Equations with Arbitrary Anisotropic Grids

Lennard Kamenski[1]

There is a concern that anisotropic grids containing elements of high aspect ratio may dramatically increase the conditioning of the finite element equations. Classic results for isotropic adaption are not useful for anisotropic grids since they lead to an excessive overestimation of the real condition number of the stiffness matrix. Thus, a new analysis was necessary. This talk presents an overview of the available results as well as recent achievements, which show that the conditioning of the finite elements is not necessarily as bad as it is generally assumed.

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Numerical Simulation of the Interface Moving and Growing Problems Using Small Mesh Deformation

Huidong Yang[1] Ulrich Langer[2]

In this talk, we will present a simple cutting method in solving the moving and growing interface problems in 3D. This new method is able to resolve large displacement or deformation of the immersed object, by combining Arbitrary Lagrangian-Eulerian method with only small local mesh deformation defined on the reference domain, that is decomposed into the so-called macro elements. The arising linear system of equations after temporal and spatial discretization is solved by either the all-at-once or the segregated methods, that are based on the algebraic multigrid methods.

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Finite Elements for Mechanochemical Pattern Formation

Felix Brinkmann[1] Thomas Richter[2] Moritz Mercker[3] Anna Marciniak-Czochra[4]

In this talk a finite element method for mechanochemical pattern formation will be presented. A biological application of this prototypic model is embryonic development of fertilized cells.

We model biological tissues using the hyperelastic Saint Venant-Kirchhoff model. The growth processes are modeled by splitting the deformation gradient into an active part and an elastic response. The active part depends on the concentration of signaling molecules, which are modeled by an reaction-diffusion equation.

Evolving patterns are reinforced by a feedback mechanism since the experimental observations show that biological cells react to stress and to the change of their shape.
We will present a mechanism using stress as well as a mechanism using strain which is stable under different initial conditions.

Finally, implementation details such as oscillating growth and rotating solutions will be addressed. Large problems, in particular in 3D, are solved with a parallel multi grid solver of the software library Gascoigne 3D.

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Viscoelasticity with Internal State Variables at Large Strain Deformations

Hansjörg Schmidt[1] Arnd Meyer[2]

Polymers, soft tissues and other materials can possess viscoelastic material behaviour, i. e. the deformation depends not solely on the amplitude of the external forces, but also on their rate. Also, this materials can reduce stresses or increase strains if the external forces remain constant over time. Of particular interest are the residual stresses in parts produced by injection moulding. These residual stresses can cause unfavourable warpage but, on the other hand, can also increase the strength of the material in some load cases. We present a theory of viscoelasticity at large strain deformations using internal state variables. Starting from a non-linear weak formulation, we examine the viscoelastic stress-strain relation, the evolution equation of the internal variable and the linearised system of equations. Some numerical results illustrate the viscoelastic behaviour.

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Full Vectorial Finite Element Beam Propagation Method for Simulation of Depolarization in Laser Amplifiers

Rainer Hartmann[1] Christoph Pflaum[2]

We developed a complex physical model for simulating laser amplifiers to numerically analyze birefringence effects. This model will be useful in designing and optimizing solid-state, high power, continuous wave laser amplifier. Our analysis includes pump configuration, thermal lensing effects, birefringence, and beam propagation in the laser amplifier. In particular, temperature, deformation, and stress inside the laser crystal were calculated using a three-dimensional finite element analysis (FEA). The pump configuration for the active medium is simulated using a three-dimensional ray tracing. The process of amplification takes into account beam propagation, gain saturation, and depolarization. The thermal input to the active medium induces thermal lensing, deformation, and birefringence. The effects of thermal lensing and deformation on beam propagation can be well approximated using ray transfer matrix approximations. However, including the effects of depolarization requires a more accurate simulation tool. Depolarization is mainly influenced by the anisotropic stress induced change of the refractive index. The beam propagation method (BPM) has also been extended to be applicable to simulate the propagation of light in general anisotropic materials. Our simulations show the depolarization of a linearly polarized electromagnetic wave in a cylindrical laser crystal. These simulations were performed using a three-dimensional full vectorial BPM. The laser crystal uses a block structured finite element discretization for the transverse plane based on quadrangles. The beam is propagated in direction z using the Crank-Nicolson method that is coupled with an explicit stepping for the laser rate equations. Reflections of outgoing waves are omitted by a perfect matching layer. The two-dimensional propagation operator of the BPM requires small mesh sizes in order to accurately model the phase shift of the electromagnetic field. Therefore, we implemented a third order spline interpolation for the refinement of the refractive index that was obtained from the three-dimensional structural analysis.

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Efficient Simulation of Short Fibre Reinforced Composites

Rolf Springer[1] Arnd Meyer[2]

Lightweight structures became more and more important over the last years. One special class of such structures are short fibre reinforced composites, produced by injection moulding. To avoid expensive experiments for testing the mechanical behaviour of these composites proper material models are needed. Thereby, the stochastic nature of the fibre orientation is the main problem.

In this talk we look onto the simulation of such materials in a linear thermoelastic setting. So, we use the stress-strain relation

$$\sigma = \mathfrak{C} : (\varepsilon - (\theta - \theta_0)\mathbf{T}), \tag{1}$$

with a fourth order material tensor $\mathfrak{C}$, a second order thermal expansion tensor $\mathbf{T}$, the temperature difference $(\theta - \theta_0)$, and the second order linearised strain tensor ε.
The needed material properties can be described by averaging the material properties of transversely isotropic materials. In this talk we will present how this can be done in an intuitive way. We also show, how these idea can be used for the description of the arising stresses within such composites.
Furthermore, we look onto equation (1) in a time dependent setting, i.e. θ is described by

$$c_\rho \rho \frac{\partial \theta}{\partial t} - \nabla(\kappa \cdot \nabla \theta) = \Theta,$$

with the material density ρ, the specific heat capacity c_ρ and the symmetric second order heat conduction tensor κ. Here, κ also depends on the stochastic fibre orientation. For this setting, we will present some numerical results.

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Discretization of Control Constrained Optimal Control Problems with Higher Order Finite Elements

Gerd Wachsmuth[1] Arnd Rösch[2]

In this talk we consider the optimal control of an elliptic PDE subject to pointwise control constraints. We discretize the problem with higher order finite elements in order to achieve fast convergence with respect to the mesh width h.

We propose a discretization of the optimal control problem which uses lumping for the mass matrices. This scheme has the advantage that "optimization" and "discretization" commute. We provide a-priori error estimates. In the case of P^2 finite elements, we achieve the order h^2 for the L^2 error in the state and the adjoint state on polygonal convex domains with uniform meshes. Using meshes which are slightly refined at the interface of the active and inactive set, we get convergence of order h^3, where h is the mesh-width in the bulk of the domain. We also obtain convergence order h^3 for the optimal control in the nodes of the discretization and for a post-processed control.

The theoretical findings are confirmed by numerical experiments.

This is joint work with Arnd Rösch (Universität Duisburg-Essen).

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Numerical Analysis for the Optimal Control of the Full Time-Dependent Maxwell Equations

Vera Bommer[1] Irwin Yousept[2]

In this talk an optimal control problem of the full time-dependent Maxwell equations and its numerical analysis are presented. We aim at finding the optimal current density and its time-dependent amplitude which steer the electric and magnetic fields to the desired ones. The mathematical analysis of the control problem which includes existence and regularity results is briefly discussed. Then, by choosing the lowest order edge elements of Nédélecs first family for the current density and continuous $\mathbb{P}_1$-elements for the amplitude a finite element approximation of the control is established. Further, the Maxwell equations, that have a first order hyperbolic coupled structure, are discretized by mixed finite elements based on piecewise constant elements for the electric field and lowest order edge elements of Nédélecs first family for the magnetic field. The time discretization is obtained via a Crank-Nicolson scheme. We present preliminary theoretical results on the finite element approximation of the optimal control problem including some 3D numerical examples. This is joint work with Irwin Yousept.

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A-priori L^p-Error Analysis for the Obstacle Problem

Constantin Christof[1]

This talk is concerned with the accuracy that is obtained when piecewise linear finite elements are used to approximate the solution of the unilateral obstacle problem

$$\min \int_\Omega \nabla v \cdot \nabla v \, \mathrm{d}x - 2 \langle f, v \rangle$$
$$\text{s.t. } v \in \{w \in H_0^1(\Omega) : \; w \geq \psi \text{ a.e.}\}.$$

The starting point of our investigation is a generalization of Céa's lemma which goes back to Falk and allows to derive a priori estimates for the H^1-error between the exact and the approximate solution. We then move on to some lesser known results by Mosco which can be utilized to extend the classical Nitsche Trick to one-dimensional problems with inequality constraints. After that, it is demonstrated by means of a counter example that a general a priori error estimate of the form

$$\|u - u_h\|_{L^q} \leq C\, h^2$$

for some $1 \leq q \leq \infty$ cannot be obtained for the obstacle problem unless the obstacle ψ is assumed to possess $W^{2,\infty}$-regularity. Using a discrete maximum principle we subsequently derive error estimates in the L^∞-norm which are optimal at least in the one-dimensional case. Lastly, we will discuss some questions which remain open in the multivariate setting.

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Non-Conforming Galerkin Finite Element Methods for Local Absorbing Boundary Conditions of Higher Order

Kersten Schmidt[1] Julien Diaz[2] Christian Heier[3]

Local absorbing boundary conditions are used to mimick the solution in presence of an infinite exterior in diffusion problems or time-harmonic scattering problems, in highly conducting bodies or thin layers. We consider Dirichlet-to-Neumann boundary conditions involving higher tangential derivatives (see an analysis in K. Schmidt and C. Heier, ESAIM Math. Model. Numer. Anal., 49(1): 257–273, 2015). If only second derivatives are present, *i.e.*, for the Neumann, Robin and Wenttzel conditions, and the boundary is smooth enough, we can incorporate the condition in usual piecewise continuous finite element methods. For higher derivatives trial and test functions with higher continuity (at least) along the boundary or auxilliary unknowns may be used. We propose as an alternative nonconforming interior penalty finite element methods for usual continuous finite element spaces in additional terms on the nodes of the boundary appear. For fourth order PDEs a similar approach has been introduced in S. Brenner and L.-Y. Sung, J. Sci. Comput. 22-23, 84–118, 2005. We will present well-posedness results and *a-priori* h-convergence error estimates for uniform polynomial degrees. The theoretical convergence results are validated by a series of numerical experiments.

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Non-Symmetric Coupling of FVM and BEM

Günther Of[1] Christoph Erath

We present the analysis of the non-symmetric coupling of a finite volume method and the boundary element method to solve a transmission problem of a diffusion convection reaction problem in the interior domain and a diffusion problem in the exterior. The discrete system maintains naturally local conservation, and allows for an upwind scheme to guarantee stability for convection dominated problems. We present numerical examples to demonstrate the benefits of the presented approach.

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Analysis and Numerics of a Generalised Steklov Eigenvalue Problem

Marco Zank[1] Olaf Steinbach[2]

In this talk we consider the generalised Steklov eigenvalue problem

$$-\Delta u = 0 \text{ in } \Omega,$$
$$\frac{\partial}{\partial n} u = \mu F u \text{ on } \partial\Omega =: \Gamma$$

for a given operator $F\colon\ H_*^{\frac{1}{2}}(\Gamma) \to H_*^{-\frac{1}{2}}(\Gamma)$ with the spaces

$$H_*^{-\frac{1}{2}}(\Gamma) := \left\{ w \in H^{-\frac{1}{2}}(\Gamma)\colon\ \langle w, \mathbf{1} \rangle_\Gamma = 0 \right\},$$
$$H_*^{\frac{1}{2}}(\Gamma) := \left\{ v \in H^{\frac{1}{2}}(\Gamma)\colon\ \langle w_{\mathrm{eq}}, v \rangle_\Gamma = 0 \right\}$$

and with the natural density $w_{\mathrm{eq}} := V^{-1}\mathbf{1}$ where V^{-1} is the inverse simple layer operator. The generalised Steklov eigenvalue problem is examined for the inverse simple layer operator $F = V^{-1}$ and for the hypersingular boundary integral operator $F = D$.

Spectral values of the underlying operators are linked to the spectral values of the operator $\frac{1}{2}I + K$ with the double layer operator $K\colon\ H^{\frac{1}{2}}(\Gamma) \to H^{\frac{1}{2}}(\Gamma)$. We can also find a representation of the contraction rate c_K of the operator $\frac{1}{2}I + K$. Further, the existence of Steklov eigenfunctions and Steklov eigenvalues is proved for domains providing that the double layer operator K is compact.

At the end of the talk numerical examples will be presented not only for a smooth boundary Γ but also for a boundary Γ with corners.

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An Interpolation-Based fast Multipole Method for Higher Order Boundary Elements on Parametric Surfaces

Jürgen Dölz[1] Helmut Harbrecht[2] Michael Peters[3]

We propose a black-box higher order fast multipole method for solving boundary integral equations on parametric surfaces in three dimensions. Such piecewise smooth surfaces are the topic of recent studies in isogeometric analysis. Due to the exact surface representation, the rate of convergence of higher order methods is not limited by approximation errors of the surface. An element-wise clustering yields a balanced cluster tree and an efficient numerical integration scheme for the underlying Galerkin method. By performing the interpolation for the fast multipole method directly on the reference domain, we reduce the cost complexity in the polynomial degree by one order. This gain is independent of the application of either $\mathcal{H}$- or $\mathcal{H}^2$-matrices. In fact, we point out several simplifications in the construction of $\mathcal{H}^2$-matrices, which are a by-product of the surface representation.

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Maximum Norm Estimates for Neumann BVP on Graded Meshes

Sergejs Rogovs[1] Thomas Apel [2] Johannes Pfefferer[3]

This talk deals with the numerical computation of the numerical solution of boundary value problems with Neumann boundary conditions in polygonal domains using the finite element method. Due to the singularities in the neighborhood of corners of the domain, the convergence rate of the numerical solution can be lower than in case of smooth domains. As a remedy one can use local mesh refinement near the singular points and in order to prove optimal error estimates regularity results in weighted Sobolev spaces are exploited. In such a case the convergence rate of $|\ln h|^{3/2}h^2$ using piecewise linear ansatz functions can be shown. Similar results for boundary value problems with Dirichlet boundary conditions were obtained by Th. Apel, A. Rösch and D. Sirch.

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Grad-Div Stabilisation on S-Type Meshes for the Oseen Problem

Katharina Höhne[1] Sebastian Franz[2] Gunar Matthies[3]

We consider the stationary Oseen equations

$$\begin{aligned} -\varepsilon\Delta\mathbf{u} + (\mathbf{b}\cdot\nabla)\mathbf{u} + c\mathbf{u} + \nabla p &= \mathbf{f} && \text{in } \Omega = (0,1)^2,\\ \operatorname{div}\mathbf{u} &= 0 && \text{in } \Omega,\\ \mathbf{u} &= 0 && \text{on } \Gamma = \partial\Omega, \end{aligned}$$

where $0 < \varepsilon \ll 1$, $\mathbf{b} \in W^1_\infty(\Omega)^2$ with $\operatorname{div}\mathbf{b} = 0$ and $L_\infty(\Omega) \ni c \geq 0$. The Oseen problem can be seen as a linearisation of the Navier-Stokes equations. For this problem, we formulate a discrete problem on S-type meshes. Additionally, we make use of a grad-div stabilisation term $\gamma(\operatorname{div} u, \operatorname{div} v)$. With the help of an assumption for the structure of the velocity u, we show convergence results of the type

$$|||(u-u_h, p-p_h)||| \leq C\left(1+\frac{1}{\beta}\right)(h + l + N^{-1}\max|\psi'|)^k,$$

where β depends on the inf-sup constant β_0. The parameters h, l, N are the grid sizes and ψ is the special function for the S-type mesh. The experiments were done with $Q_k \times Q_{k-1}$ elements and $Q_k \times P^{\text{disc}}_{k-1}$ elements of arbitrary order k. Furthermore, we investigate numerically the inf-sup constant β_0 and its dependence on the mesh.

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Singularly Perturbed Turning Point Problems in one Dimension

Simon Becher[1]

We consider singularly perturbed boundary value problems with interior and/or boundary turning points in one dimension. It is well known that the solution of such problems often exhibits certain interior and/or boundary layers.

We give an overview about the a priori estimates of the solution for the several types of turning points. Furthermore, we present some numerical experiments for solving the problem on layer-adapted meshes with FEM or upwind-FDM.

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The Current Landscape of Energy A-posteriori Error Estimators

Martin Vohralík[1]

In this talk, we try to review the different existing versions of a posteriori estimators in the context of energy error control for the Laplace equation. We in particular consider residual-based, averaging, hierarchical, functional, equilibrated residual, geometric (interpolation), and equilibrated flux (constitutive relation) estimators. We examine whether they lead to a guaranteed error upper bound, local efficiency, asymptotic exactness, robustness (with respect to the data and the polynomial degree of the approximation), and whether they have low evaluation cost. We also discuss if they are problem-dependent or not, implicit (local solves necessary) or completely explicit (directly prescribed from the approximate solution), ensure guaranteed maximal (local) overestimation, and enable to distinguish the different error components. Conforming, nonconforming, discontinuous Galerkin, and mixed finite element discretizations are considered. Computational examples, also for more involved unsteady and nonlinear problems, are presented for the equilibrated flux estimators.

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Reliable Averaging for the Primal Variable in the Courant FEM and Hierarchical Error Estimators on Red-Refined Meshes

Martin Eigel[1] Carsten Carstensen[2]

A hierarchical a posteriori error estimator for the first-order finite element method on a red-refined triangular mesh is presented for the 2D Poisson model problem. Reliability and efficiency with an explicit constant can be proved for triangulations with inner angles smaller than or equal to $\pi/2$. The error estimator does not rely on any saturation assumption and is valid even in the pre-asymptotic regime on arbitrarily coarse meshes. The evaluation of the estimator is a simple post-processing of the piecewise linear FEM without any extra solve. It is a striking observation that arbitrary local averaging of the primal variable leads to a reliable and efficient error estimation. Numerical experiments illustrate the performance of the proposed a posteriori error estimator for computational benchmarks.

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Variational Localizations of Error Estimators with Application to Complex Multiphysics Problems

Thomas Richter[1] Thomas Wick[2]

Focus of this presentation will be a localization technique for goal oriented error estimators. The resulting local error indicators can be used for local mesh adaptivity. The simple idea behind the localization is to introduce a partition of unity into the variational residual. The local error indicators can be computed without jumps over element edges and also without strong residuals. While edge jumps can be cumbersome to evaluate, strong residuals may not even be available for the adjoint systems of complex problems. For elliptic problems, we can show, that the sum of local indicators is bound by local products of primal and adjoint energy norm errors.

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Variational Multiscale Stabilization and the Exponential Decay of Fine-scale Correctors

Daniel Peterseim[1]

This lecture addresses the variational multiscale stabilization of standard finite element methods for partial differential equations that exhibit multiscale features. The stabilization is of Petrov-Galerkin type with a standard finite element trial space and a problem-dependent test space based on pre-computed fine-scale correctors. The exponential decay of these correctors and their localisation to local cell problems is rigorously justified. The stabilization technique eliminates scale-dependent pre-asymptotic effects as they appear in standard finite element approximations of highly oscillatory problems, e.g., the poor L^2 approximation in homogenization problems and the pollution effect in high-frequency acoustic scattering.

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Hyperelastic Laminates, Composite Voxels and FFT-based Homogenization at Finite Strains

Felix Ospald[1] Matthias Kabel[2] Matti Schneider[3]

Today, many industrial applications involve complex materials with a heterogeneity on the microscopic scale. For the prediction of the effective behavior of these materials, homogenization methods are readily applied. However, especially for nonlinear material laws these methods are heavily demanding from the computational perspective, often pushing commercial FEM software on workstations to its limits.

In contrast to boundary conforming FEM, FFT-based homogenization methods operate on regular voxel grids and cannot resolve interfaces exactly in general, therefore requiring excessively high resolutions for accurate results. In this talk, we study hyperelastic laminates and associate their effective properties to voxels containing interfaces, extending previous ideas successfully applied in the framework of linear elasticity [1].

We demonstrate that furnishing interface voxels with appropriately rotated effective hyperelastic properties of a two-phase laminate significantly enhances both the local solution quality and the accuracy of the computed effective elastic properties, with only a small computational overhead compared to using classical FFT-based homogenization.

The results are also readily applicable for non-boundary conforming FEM, like micro finite element analysis [2].

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Multiscale Mixed Finite Elements

Fredrik Hellman[1] Patrick Henning[2] Axel Målqvist[3]

In this talk, we propose a mixed finite element method for solving elliptic multiscale problems. It does not rely on structural assumptions of the problem (e.g., periodicity) and converges independently of the regularity of the solutions. A low dimensional multiscale mixed finite element space with high approximation properties based on the Raviart–Thomas finite element spaces is constructed. This space can be used to solve the original saddle point problem efficiently. The method requires to solve many local problems in patches around the elements of a coarse grid. These computations can be perfectly parallelized and are cheap to perform. The applicability of the method is verified by a variety of numerical experiments.

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Model Reduction Techniques for Numerical Homogenization

Assyr Abdulle[1]

Many multiscale methods rely on the construction of localized basis or on local recovery of some macroscopic parameters to set up an efficient macroscopic scheme. In this talk we discuss the design and analysis of multiscale methods combined with reduced order modeling techniques such as the reduced basis method. We will present reduced basis numerical homogenization methods for quasilinear problems and Stokes problems with multiple scales. Multiscale reduced basis method for problems without scale separation based on local orthogonal decompositions will also be discussed. This talk is based upon a series of joint works with various collaborators (see the references below).

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An Adaptive GMsFEM for High-Contrast Flow Problems

Guanglian Li[1] Eric T. Chung[2] Yalchin Efendiev[3]

In this paper, we derive an a-posteriori error indicator for the Generalized Multiscale Finite Element Method (GMsFEM) framework. This error indicator is further used to develop an adaptive enrichment algorithm for the linear elliptic equation with multiscale high-contrast coefficients. We consider two kinds of error indicators where one is based on the L^2-norm of the local residual and the other is based on the weighted H^{-1}-norm of the local residual where the weight is related to the coefficient of the elliptic equation. We show that the use of weighted H^{-1}-norm residual gives a more robust error indicator which works well for cases with high contrast media. The convergence analysis of the method is given. This is a joint work with Dr. Eric T. Chung (CUHK) and Dr. Yalchin Efendiev (TAMU).

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Numerical Homogenization for the Wave Equation

Patrick Henning[1] Assyr Abdulle[2]

In this talk we discuss the issues arising for the wave equation with a continuum of scales and why it is challenging to construct suitable Finite Element spaces for solving it efficiently. In this talk we propose a corresponding multiscale method which is capable of constructing accurate L2-approximations. The proposed method does not require any assumptions on space regularity or scale-separation and it is formulated in the framework of the Localized Orthogonal Decomposition (LOD). The convergence rates vary between linear convergence and third order convergence depending on the considered initial values.

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A-posteriori Error Control for Stationary Coupled Bulk-Surface Equations

Rüdiger Müller[1] Martin Eigel[2]

We consider a system of two coupled elliptic equations, one defined on a bulk domain $\Omega \subset \mathbb{R}^d$, $d \in \{2,3\}$ with piecewise smooth boundary and the second one on the boundary surface $\Gamma := \partial\Omega$ that is decomposed into a finite set of patches $\{\Gamma^i\}_{i=1}^N$. Such coupled reaction diffusion processes in the bulk and on the surface have recently attracted interest from an analytical point of view [2] and in different application areas such as biology and chemistry, see e.g. [3]. We seek the solution $u : \Omega \to \mathbb{R}$ and $v : \Gamma \to \mathbb{R}$ of the stationary coupled diffusion-reaction problem

$$-\Delta u + u = f \qquad \text{in } \Omega, \tag{1a}$$
$$(\alpha u - \beta v) + \partial_n u = 0 \qquad \text{on } \Gamma, \tag{1b}$$
$$-\underline{\Delta}_\Gamma v + v + \partial_n u = g \qquad \text{on } \Gamma, \tag{1c}$$
$$\underline{\nabla}_{\Gamma^i} v \cdot n^i + \underline{\nabla}_{\Gamma^j} v \cdot n^j = 0 \qquad \text{on } \partial\Gamma^i \cap \partial\Gamma^j\ . \tag{1d}$$

A-priori analysis for domains with smooth boundary has been established in [2] and a-posteriori analysis for a pure surface problem can be found in [1]. Here, a-posteriori error control is proved. We derive a fully computable residual estimator that takes into account the approximation errors due to discretization with lowest order continuous finite elements in space as well as errors due to polyhedral approximation of the surface. An adaptive refinement algorithm is described which controls the overall error. Numerical experiments illustrate the performance of the a-posteriori error estimator and the proposed adaptive algorithm.

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Adaptive Wavelet BEM

Manuela Utzinger[1]

In this talk, we will consider the adaptive wavelet boundary element method for the solution of boundary integral equations in three dimensions. Partial differential equations are frequently encountered in science and engineering, some of which can be formulated as boundary integral equations. This method, on one hand, allows us to reduce the dimensionality of the problem (3d to 2d), at the same time it is a possibility to deal with the infinite expansion of the domain, in case we consider an exterior problem.

For a domain with small geometrical features, or one containing corners and edges, we require a strong refinement in certain small parts of the geometry. In such cases, uniform refinement may not be an option any more (e.g. huge systems), making it necessary to have an adaptive approach at hand.

Even though the dimensionality of the underlying problem is already reduced drastically by working only on the boundary and using adaptivity, the involved matrix still is densely populated. Methods like panel clustering, hierarchical matrices or the adaptive cross approximation allow to reduce the complexity to log-linear or even linear cost. We will use wavelet compression which results in linear cost.

We will present an algorithm for the adaptive solution of boundary integral equations. In particular, we will elaborate on the efficient implementation of this algorithm. Finally, we will give various numerical examples involving the Laplace and the Helmholtz equation on different geometries.

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Adaptive Optimal Control of Contact Problems

Korinna Rosin[1] Christian Meyer[2] Andreas Rademacher[3]

In this talk, we discuss optimal control of contact problems, for instance obstacle or Signorini problems. To employ gradient based algorithms the underlying problem is regularized and then discretized by finite elements. An approach to estimate the arising regularization, discretization and numerical errors based on the dual weighted residual (DWR) method is presented. A similar idea was recently introduced by Meyer, Rademacher and Wollner for the obstacle problem. However, instead of estimating the different errors by separate approaches, the DWR method is directly applied to estimate the total error. The resulting terms of the estimate can be assigned to the different error sources. Finally, we utilize the estimator in an adaptive refinement strategy balancing regularization and discretization errors. Numerical results substantiate the theoretical findings.

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Discretization of Elliptic Differential Equations Using Sparse Grids and Prewavelets

Christoph Pflaum[1] Rainer Hartmann[2]

Sparse grids can be used to discretize elliptic differential equations of second order on a d-dimensional cube. Using the Galerkin discretization, one obtains a linear equation system with $O(N(logN)^{d-1})$ unknowns. The corresponding discretization error is $O(N^{-1}(logN)^{d-1})$ in the H^1-norm. A major difficulty in using this sparse grid discretization is the complexity of the related stiffness matrix. As a consequence only PDE's with constant coefficients can be efficiently be discretized using the standard sparse grid discretization with $d > 2$. To reduce the complexity of the sparse grid discretization matrix, we apply prewavelets. This simplifies the implementation of the corresponding algorithms. Furthermore, we present a new sparse grid discretization for the discretization of elliptic differential equations with variable coefficients. This discretization utilizes a semi-othogonality property. The convergence rate and stability of the discretization is proven for arbitrary dimensions d.

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Local Refinement for T-Splines in 2D and 3D

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T-splines are a generalization of tensor-product B-splines to non-uniform meshes. They have been introduced as a free-form geometric technology in the Computer-Aided Design community and have therefore caught much attention in Isogeometric Analysis, particularly with regard to constructing an Adaptive Finite Element Method that directly uses data structures from CAD applications. We present an efficient adaptive refinement procedure that preserves analysis-suitability of the mesh, this is, the linear independence of the T-spline functions. We prove analysis-suitability of the overlays and boundedness of their cardinalities, nestedness of the generated T-spline spaces, and linear computational complexity of the refinement procedure in terms of the number of marked and generated mesh elements. In addition, we generalize the algorithm to the three-dimensional case.

References:

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FEM for NURBS Surface Shells

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The application of shell theories in FEM requires a description of the shell midsurface geometry and its derivatives. Modern computer aided design software uses non-uniform rational B-splines for geometry description. Therefore, it is natural to incorporate NURBS surfaces in an FEM code.

In our talk we give some insights on how to implement FEM for Naghdi shells. A specialty is the incorporation of tangential vectors with continuous directions, but jumping lengths. Such vectors may occur when NURBS surfaces are glued together.

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A Conjugate Direction Method for Linear Systems in Banach Spaces

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The well known conjugate gradient method for the solution of self-adjoint linear systems $Ax = b$ with positive definite operator A makes extensive use of the Hilbert space structure of the underlying space. We investigate an extension to the Banach space setting, in which the Riesz isomorphism has to be replaced by a duality mapping. Due to the nonlinearity of the latter, the short term recursion and the conjugacy of search directions cannot be maintained simultaneously. We address the well-posedness of the proposed iteration and its global convergence. Error bounds and stopping criteria are presented as well. The behavior of the method is demonstrated by means of numerical examples.

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- Tea, coffee, soft drinks and snacks during breaks.
- The conference dinner on Monday.

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