

A Conjugate Direction Method for Linear Systems in Banach Spaces

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The well known conjugate gradient method for the solution of self-adjoint linear systems $Ax = b$ with positive definite operator A makes extensive use of the Hilbert space structure of the underlying space. We investigate an extension to the Banach space setting, in which the Riesz isomorphism has to be replaced by a duality mapping. Due to the nonlinearity of the latter, the short term recursion and the conjugacy of search directions cannot be maintained simultaneously. We address the well-posedness of the proposed iteration and its global convergence. Error bounds and stopping criteria are presented as well. The behavior of the method is demonstrated by means of numerical examples.

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