

# Reliable Averaging for the Primal Variable in the Courant FEM and Hierarchical Error Estimators on Red-Refined Meshes

Martin Eigel<sup>1</sup> Carsten Carstensen<sup>2</sup>

A hierarchical a posteriori error estimator for the first-order finite element method on a red-refined triangular mesh is presented for the 2D Poisson model problem. Reliability and efficiency with an explicit constant can be proved for triangulations with inner angles smaller than or equal to  $\pi/2$ . The error estimator does not rely on any saturation assumption and is valid even in the pre-asymptotic regime on arbitrarily coarse meshes. The evaluation of the estimator is a simple post-processing of the piecewise linear FEM without any extra solve. It is a striking observation that arbitrary local averaging of the primal variable leads to a reliable and efficient error estimation. Numerical experiments illustrate the performance of the proposed a posteriori error estimator for computational benchmarks.

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<sup>1</sup> Weierstrass Institute, Nonlinear Optimisation and Inverse Problems, Berlin,  
[eigel@wias-berlin.de](mailto:eigel@wias-berlin.de)

<sup>2</sup> Humboldt-Universität zu Berlin, Institut für Mathematik,  
[cc@math.hu-berlin.de](mailto:cc@math.hu-berlin.de)