Analysis and Numerics of a Generalised Steklov Eigenvalue Problem

Marco Zank[1] Olaf Steinbach[2]

In this talk we consider the generalised Steklov eigenvalue problem

$$-\Delta u = 0 \text{ in } \Omega,$$
$$\frac{\partial}{\partial n} u = \mu F u \text{ on } \partial\Omega =: \Gamma$$

for a given operator $F\colon\ H_*^{\frac{1}{2}}(\Gamma) \to H_*^{-\frac{1}{2}}(\Gamma)$ with the spaces

$$H_*^{-\frac{1}{2}}(\Gamma) := \left\{ w \in H^{-\frac{1}{2}}(\Gamma)\colon\ \langle w, \mathbf{1}\rangle_\Gamma = 0 \right\},$$
$$H_*^{\frac{1}{2}}(\Gamma) := \left\{ v \in H^{\frac{1}{2}}(\Gamma)\colon\ \langle w_{\text{eq}}, v\rangle_\Gamma = 0 \right\}$$

and with the natural density $w_{\text{eq}} := V^{-1}\mathbf{1}$ where V^{-1} is the inverse simple layer operator. The generalised Steklov eigenvalue problem is examined for the inverse simple layer operator $F = V^{-1}$ and for the hypersingular boundary integral operator $F = D$.

Spectral values of the underlying operators are linked to the spectral values of the operator $\frac{1}{2}I + K$ with the double layer operator $K\colon\ H^{\frac{1}{2}}(\Gamma) \to H^{\frac{1}{2}}(\Gamma)$. We can also find a representation of the contraction rate c_K of the operator $\frac{1}{2}I + K$. Further, the existence of Steklov eigenfunctions and Steklov eigenvalues is proved for domains providing that the double layer operator K is compact.

At the end of the talk numerical examples will be presented not only for a smooth boundary Γ but also for a boundary Γ with corners.

[1] Graz University of Technology, Institute of Computational Mathematics, Graz, Austria, zank@math.tugraz.at

[2] Graz University of Technology, Institute of Computational Mathematics, Graz, Austria, o.steinbach@tugraz.at