Grad-Div Stabilisation on S-Type Meshes for the Oseen Problem

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We consider the stationary Oseen equations

$$\begin{aligned} -\varepsilon\Delta\mathbf{u} + (\mathbf{b}\cdot\nabla)\mathbf{u} + c\mathbf{u} + \nabla p = \mathbf{f} \qquad & \text{in } \Omega = (0,1)^2, \\ \operatorname{div}\mathbf{u} = 0 \qquad & \text{in } \Omega, \\ \mathbf{u} = 0 \qquad & \text{on } \Gamma = \partial\Omega, \end{aligned}$$

where $0 < \varepsilon \ll 1$, $\mathbf{b} \in W^1_\infty(\Omega)^2$ with $\operatorname{div}\mathbf{b} = 0$ and $L_\infty(\Omega) \ni c \geq 0$. The Oseen problem can be seen as a linearisation of the Navier-Stokes equations. For this problem, we formulate a discrete problem on S-type meshes. Additionally, we make use of a grad-div stabilisation term $\gamma(\operatorname{div} u, \operatorname{div} v)$. With the help of an assumption for the structure of the velocity u, we show convergence results of the type

$$|||(u-u_h, p-p_h)||| \leq C\left(1+\frac{1}{\beta}\right)(h+l+N^{-1}\max|\psi'|)^k,$$

where β depends on the inf-sup constant β_0. The parameters h, l, N are the grid sizes and ψ is the special function for the S-type mesh. The experiments were done with $Q_k \times Q_{k-1}$ elements and $Q_k \times P^{\text{disc}}_{k-1}$ elements of arbitrary order k. Furthermore, we investigate numerically the inf-sup constant β_0 and its dependence on the mesh.

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