Robust Arbitrary Order Mixed Finite Element Methods for the Incompressible Stokes Equations with Pressure Independent Velocity Errors

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Standard mixed finite element methods for the incompressible Navier–Stokes equations that relax the divergence constraint are not robust against large irrotational forces in the momentum balance and the velocity error depends on the continuous pressure. This robustness issue can be completely cured by using divergence-free mixed finite elements which deliver pressure-independent velocity error estimates. However, the construction of H^1-conforming, divergence-free mixed finite element methods is rather difficult. Instead, we present a novel approach for the construction of arbitrary order mixed finite element methods which deliver pressure-independent velocity errors. The approach does not change the trial functions but replaces discretely divergence-free test functions in some operators of the weak formulation by divergence-free ones. This modification is applied to inf-sup stable conforming and nonconforming mixed finite element methods of arbitrary order in two and three dimensions. Optimal estimates for the incompressible Stokes equations are proved for the H^1 and L^2 errors of the velocity and the L^2 error of the pressure. Moreover, both velocity errors are pressure-independent, demonstrating the improved robustness. Several numerical examples illustrate the results.

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